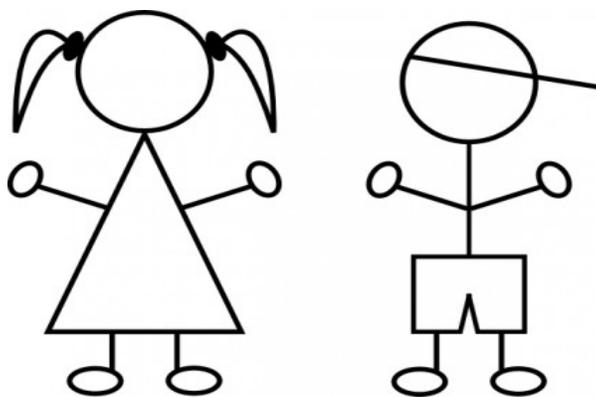


# **Addition & Subtraction Fact Strategies**



**Wichita Public Schools**  
**Curriculum and Instructional Design**  
**Mathematics**

**Revised 2014**  
**KCCRS version**

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# Research Connections

## Where Do Fact Strategies Fit In?

Adapted from Randall Charles

Fact strategies are considered a crucial second phase in a three-phase program for teaching students basic math facts.

The first phase is **concept learning**. Here, the goal is for students to understand the meanings of multiplication and division. In this phase, students focus on actions (i.e. "groups of", "equal parts", "building arrays") that relate to multiplication and division concepts.

An important instructional bridge that is often neglected between concept learning and memorization is the second phase, **fact strategies**. There are two goals in this phase. First, students need to recognize there are clusters of multiplication and division facts that relate in certain ways. Second, students need to understand those relationships. These lessons are designed to assist with the second phase of this process. If you have students that are not ready, you will need to address the first phase of concept learning.

The third phase is **memorization** of the basic facts. Here the goal is for students to master products and quotients so they can recall them efficiently and accurately, and retain them over time.

Learning86, January

## Teaching Student-Centered Mathematics

John Van de Walle, Jennifer Bay-Williams, LouAnn Lovin, Karen Karp

When students count on their fingers or make marks in the margins they have not mastered their facts because they have not developed efficient methods of producing a fact answer based on number relationships and reasoning. Drilling inefficient methods does not produce mastery!

Over many years, research supports the notion that basic fact mastery is dependent on the development of reasoning strategies. These reasoning strategies are essential to fact development.

*Guided invention* is an effective research-informed method for fact mastery. Teachers should design sequenced tasks and problems that will promote students' invention of effective strategies. Then, students need to clearly articulate these strategies and share them with peers. This sharing is often best carried out in think-alouds, in which student talk through the decisions they made and share counterexamples.

## Effective Drill and Practice

1. Avoid inefficient practice. Practice will strengthen strategies and make them increasingly automatic. Do not subject any student to fact drills unless the student has developed efficient strategies for the facts being practiced.
2. Individualize practice. Different students will bring different number tools to the task and will develop strategies at different rates. This means there are few drills that are likely to be efficient for a full class at any given time. That is why we need to create a large number of practice activities promoting different strategies and addressing different collections of facts.
3. Practice strategy retrieval. When students are involved in a drill exercise that is designed to practice a particular strategy, it is likely they will use that strategy. Organize the students' practice problems according to a selected strategy.

Teaching Student-Centered Mathematics: Volume 2, Van de Walle, p. 94 – 95

## Three Steps on the Road to Fluency with Basic Facts

Kim Sutton

1. Teach for Understanding
  - Multiplication
    - Repeated addition
    - Area
  - Division
    - Repeated subtraction
    - Area to length of sides
2. Teach in a meaningful sequence, then practice!

## Limitations and Risks of Timed Mathematics Tests

Jennifer Bay-Williams & Gina Kling

Timed tests offer little insight about how flexible students are in their use of strategies or even which strategies a student selects. And evidence suggests that efficiency and accuracy may actually be negatively influenced by timed testing. A study of nearly 300 first graders found a negative correlation between timed testing and fact retrieval and number sense (Henry and Brown 2008). Children who were frequently exposed to timed testing demonstrated lower progress toward knowing facts from memory than their counterparts who had not experienced as many timed tests. In fact, growing evidence suggests that timed testing has a negative impact on students (Boaler 2012, Henry and Brown 2008, Ramirez et al. 2013).

(from *Teaching Children Mathematics*, April 2014, pp 488 – 497)

## KCCRS Required Fluencies

K – 2 grade band				3 – 5 grade band		
	Conceptual Understandings that lead to Fluency: <ul style="list-style-type: none"> <li>• Addition and Subtraction (concepts, skills, problem solving)</li> <li>• Place Value</li> </ul>			Conceptual Understandings that lead to Fluency: <ul style="list-style-type: none"> <li>• Multiplication and Division of Whole Numbers and Fractions (concepts, skills, problem solving)</li> </ul>		
Required Computational/ Procedural Fluency	<b>Kindergarten</b> K.OA.5 – add/subtract within 5	<b>1<sup>st</sup> grade</b> 1.OA.6 – add/subtract within 10	<b>2<sup>nd</sup> grade</b> 2.OA.2 – add/subtract within 20 2.NBT.5 – add/subtract within 100	<b>3<sup>rd</sup> grade</b> 3.OA.7 – multiply/divide within 100 3.NBT.2 – add/subtract within 1000	<b>4<sup>th</sup> grade</b> 4.NBT.4 – add/subtract within 1,000,000	<b>5<sup>th</sup> grade</b> 5.NBT.5 – multi-digit multiplication

### Defining Fluency

Jennifer Bay-Williams & Gina Kling (from *Teaching Children Mathematics*, April 2014)

A variety of interpretations exist for what procedural fluency (in general) and basic fact fluency (specifically) mean. Fortunately, recent standards, research, and reports provide a unified vision of what these terms signify. The Common Core State Standards for Mathematics (CCSSM) document describes procedural fluency as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (CCSSI 2010, p. 6). Likewise, Baroody (2006) describes basic fact fluency as “the efficient, appropriate, and flexible application of single-digit calculation skills and . . . an essential aspect of mathematical proficiency” (p. 22). These definitions reflect what has been described for years in research and standards documents (e.g., NCTM 2000, 2006; NRC 2001) as well as CCSSM grade-level expectations related to basic facts (see table 1).

Notice that the CCSSM expectations use two key phrases; the first is to fluently add and subtract (or multiply and divide), and the second is to know from memory all sums (products) of two one-digit numbers. To assess basic fact fluency, all four tenets of fluency (flexibility, appropriate strategy use, efficiency, and accuracy) must be addressed. Additionally, assessments must provide data on which facts students know from memory. Timed tests are commonly used to supply such data—but how effective are they in doing so?

### Fluency: Simply Fast and Accurate? I Think Not!

Linda Gojak (NCTM Past-President) – from *NCTM Summing It Up*, Nov. 1, 2012  
 Our students enter school with the misconception that the goal in math is to do it fast and get it right. Do we promote that thinking in our teaching without realizing it? Do we praise students who get the right answer quickly? Do we become impatient with students who need a little more time to think? As we strive for a balance between conceptual understanding and procedural skill with mathematical practices, we must remember that there is a very strong link between the two. Our planning, our instruction, and our assessments must build on and value that connection. Fluency entails so much more than being fast and accurate!

# Addition Fact Strategies

Strategy	Strategy Description	Van de Walle
<b>Counting On- One-more-than/Two-more-than</b>	Used when adding 1 or 2 to a given number.	Vol. 1 pg 99-100 Vol. 2 pg. 79-80
<b>Facts with 0</b>	Used when one of the addends is 0 – especially helpful with story problems.	Vol. 1 pg 100 Vol. 2 pg. 79-80
<b>Doubles</b>	Adding two of the same number together, such as $2 + 2$ or $8 + 8$ .	Vol. 1 pg 101 Vol. 2 pg. 80-81
<b>Doubles + 1</b>	Finding a double hidden in the fact where one addend is one more than the other.	Vol. 1 pg. 101 Vol. 2 pg. 80-81
<b>Combinations of Ten</b>	Grouping the numbers to find expressions that would equal 10.	Vol. 1 pg. 105-106 Vol. 2 pg. 83
<b>Make Ten</b>	Use with addend of 8 or 9 building up to 10 + adding on the rest.	Vol. 1 pg. 102-103 Vol. 2 pg. 81-82
<b>Doubles + 2 Two-Apart Facts</b>	Finding a double hidden in the fact where one addend is two more than the other.	Vol. 1 pg 105 Vol. 2 pg. 83
<b>+ 9 Add 10 and take 1 away</b>	When an addend is 9, then just add 10 and take 1 away.	
<b>+ 4 Add 2 and add 2</b>	When an added is 4, add 2 and then add 2 again.	

# Subtraction Fact Strategies

Strategy	Subtraction Description	Van de Walle
<b>Think Addition</b>	Using the known addition fact to solve the subtraction problem. Ex. $13-5$ , think what goes with 5 to make 13?	Vol. 1 pg 106-107 Vol. 2 pg. 84-85
<b>Fact Families</b>	Think of the fact family to recall the missing number.	Vol. 1 pg 110-111 Vol. 2 pg. 86-87
<b>Build Up Through Ten</b>	Used when either the subtrahend or minuend is 8 or 9. Ex: $14-9$ : start with 9 and work up through 10: <i>9 and 1 is 10 and 4 more makes 5.</i>	Vol. 1 pg 108 Vol. 2 pg. 85-86
<b>Back Down Through Ten</b>	Working backward with 10 as a “bridge”. Ex. $15-6$ , Take 5 away from 15 to get to ten. Then take one more away, leaving 9.	Vol. 1 pg 109 Vol. 2 pg. 85-86

# Making Meaning for Operations (Teacher Use Only)

## Structures for Addition and Subtraction Problems

*This section is provided for teachers in order to help students develop operation sense to connect different meanings of addition and subtraction to each other. This will enable students to effectively use these operations in real-world settings. These problem structures are not intended for students but will help you as the teacher in formulating and assigning addition and subtraction tasks.*

### Math Activity: Modeling Word Problems

1. Model each of the following five problems with cubes or other counters. After you have acted out the problems with a concrete model, write an arithmetic sentence for each one.
  - a. Kris has 8 candies. She eats 3 of them. How many does she have left?
  - b. Kris has 8 candies and Marcus has 3 candies. How many more candies does Kris have?
  - c. Kris has 3 dollars. She wants to buy something that costs 8 dollars. How many more dollars does she need?
  - d. Yesterday Kris had 8 balloons. Some of them burst last evening. Today she has 3 left. How many balloons burst?
  - e. Yesterday Kris had some balloons. Today Marcus gave her 3 more balloons. Now she has 8 altogether. How many balloons did she have yesterday?
2. How are these five problems alike? How are they different?
3. What connections do you see between the five problems and the information presented on the chart, *Common Addition and Subtraction Situations*?

# Making Meaning for Operations

## Common Addition and Subtraction Situations (pg 88 in CCSS)

Shading taken from OA progression

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>Taken from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>1</sup>
<b>Put Together/ Take Apart<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
<b>Compare<sup>3</sup></b>	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

Blue shading indicates the four Kindergarten problem subtypes. Students in grades 1 and 2 work with all subtypes and variants (blue and green). Yellow indicates problems that are the difficult four problem subtypes or variants that students in Grade 1 work with but do not need to master until Grade 2.

<sup>1</sup>These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

<sup>2</sup>Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

<sup>3</sup>For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

# Addition Fact Strategies Assessment

## Directions and Analysis

As students are taking the assessment you will need to observe your students to see if they are using counting strategies (fingers, marks, verbal counting, etc.) instead of reasoning strategies. This assessment gives the teacher one “view” of what a student can do with fact strategies. This section is broken down so that each box focuses on fact strategies that are appropriate for the facts given in the box.

### Directions for the Assessment

1. Fill out the Fact Strategies View Recording Sheet so that all of your students are listed. (A clipboard or something similar should be used so that you are able to make the appropriate marks quickly.)
2. Explain to the students that they will be taking a quick assessment that will be used to assist you with their fact strategy needs.
3. Hand out the one page assessment of fact strategies to each student.
4. Ask students to write their names and the date at the top of the page.
5. Explain that they will be working in only one box at a time and that you will be making notes as they complete the box. Let students know that if they finish the box before you have asked everyone to stop, you would like for them to place their pencil down on their desk.
6. Tell students to look at Box #1. When you are certain that all students are focused on the correct box, ask them to begin. Make a check mark next to students' names on your recording sheet if you see that they are using their fingers, tally marks, manipulatives, etc. When you have made your marks, continue to the next box.
7. Ask students to look at Box #2. Have them begin and continue to do the same markings.
8. Do the same thing for all eight boxes.
9. When you are finished with the 8<sup>th</sup> box, ask the students to fill in the bottom of the paper. You may read the directions to them and allow them time to fill it out or they may fill it out on their own.

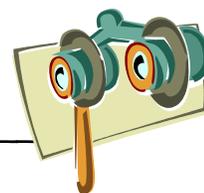
## Analysis of the Assessment

### Fact Strategies View

1. The **Fact Strategies View** page is broken apart so that the different boxes focus on strategies that are appropriate for those particular number sentences. Strategies that can be used for the examples in the box:
  - Box #1 – Counting On
  - Box #2 – Counting On (order/commutative property)
  - Box #3 – Zero
  - Box #4 – Doubles
  - Box # 5 – Doubles + 1 (Near Doubles)
  - Box #6 – Combinations of Ten
  - Box #7 – Make Ten
  - Box #8 – Doubles + 2
2. Collect the students' papers. Look at each box and check to see if students have missed 2 or more problems in that box. If they have, then they need to work on the strategies that are listed for that particular box. Make a note on the Fact Strategies View recording page.

Read what students wrote were the easiest and the hardest problems to do. This will give you an idea if the students are consciously using strategies or not, and if they are, which strategies they are most comfortable in using. Make note of this on your recording sheet.

Students that are struggling with all strategies and are counting on their fingers for all facts may need more time to work on numeration skills beginning with subitizing. This skill is discussed more fully starting on page 18. For more assistance in answering questions about these foundational skills that are necessary before fact work, please contact the Elementary Mathematics Curriculum Department.



Student's Name \_\_\_\_\_

Date \_\_\_\_\_

## Fact Strategy View (+)

#1

$8 + 1 =$

$7 + 1 =$

$6 + 2 =$

$7 + 2 =$

$5 + 2 =$

#2

$1 + 6 =$

$2 + 9 =$

$1 + 7 =$

$2 + 9 =$

$1 + 8 =$

#3

$9 + 0 =$

$4 + 0 =$

$0 + 7 =$

$5 + 0 =$

$0 + 8 =$

#4

$6 + 6 =$

$4 + 4 =$

$7 + 7 =$

$5 + 5 =$

$8 + 8 =$

#5

$5 + 6 =$

$7 + 8 =$

$6 + 7 =$

$4 + 5 =$

$3 + 4 =$

#6

$7 + 3 =$

$4 + 6 =$

$2 + 8 =$

$9 + 1 =$

$3 + 7 =$

#7

$9 + 6 =$

$4 + 9 =$

$9 + 7 =$

$5 + 9 =$

$9 + 8 =$

#8

$8 + 6 =$

$7 + 9 =$

$3 + 5 =$

$6 + 8 =$

$5 + 3 =$

Circle the box that was the easiest for you to complete.

Box #1    Box #2    Box #3    Box #4    Box #5    Box #6    Box #7    Box #8

Why was it the easiest?

---

---

Circle the box that was the hardest for you to complete.

Box #1    Box #2    Box #3    Box #4    Box #5    Box #6    Box #7    Box #8

Why was it the hardest?

---

---



## Student Discourse

Students must cognitively engage with the strategies by discussing them with their peers and their teacher. Talk is essential for full understanding about numbers and how the operations work. Encourage students to verbalize their reasoning. Encourage them to add on to another's thinking or to agree or disagree with each other. All of this must be done in a safe and supportive environment that values everyone's contributions. Two resources that we recommend to support these expectations are Classroom Discussion in Math by Suzanne Chapin and Number Talks by Sherry Parrish. The first resource provides the background for all student discourse in a mathematics classroom and comes with a DVD that shows discourse in action. The second resource explains how this can be done with a focus on mental computation and strategies. This resource also has a DVD for teachers to use.

Student discourse is a part of the expectations for the Standards for Mathematical Practice in the KCCRS.

1. *Make sense of problems and persevere in solving them.* Strategies help students see that math does make sense. There is an order and a process, but they need to talk it out with others in order to enhance and deepen their understanding.
2. *Reason abstractly and quantitatively.* Talking with peers and teachers leads thinking from the concrete to the abstract to increase their flexibility in reasoning mathematically.
3. *Construct viable arguments and critique the reasoning of others.* The heart of student discourse.
4. *Model with mathematics.* During discussion students will need to also show their thinking by writing the equations and to interpret abstract mathematics into context.
5. *Use appropriate tools strategically.* Strategies are tools for mental computation. Exploring the effectiveness of these strategies helps determine when each strategy is most appropriately used and when it is not.
6. *Attend to precision.* This standard is not only about being precise with mathematical symbols but also with math vocabulary which is essential in making thinking understood when discussing with peers.
7. *Look for and make use of structure.* Once the structure of specific strategies are understood and clarified then students are able to effectively apply appropriate strategies.
8. *Look for and express regularity in repeated reasoning.* In order to fully understand each strategy students need to see it repeatedly, and in different ways, in order to make the generalizations that are necessary for future application of that strategy.

Question Stems:

- How does this strategy work?
- What kinds of problems does this strategy help solve?
- When is this strategy not appropriate?
- How does this strategy compare with a previous strategy we explored?
- Is there more than one strategy that can be used with this problem?

# Addition Table

<b>+</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>	0	1	2	3	4	5	6	7	8	9
<b>1</b>	1	2	3	4	5	6	7	8	9	10
<b>2</b>	2	3	4	5	6	7	8	9	10	11
<b>3</b>	3	4	5	6	7	8	9	10	11	12
<b>4</b>	4	5	6	7	8	9	10	11	12	13
<b>5</b>	5	6	7	8	9	10	11	12	13	14
<b>6</b>	6	7	8	9	10	11	12	13	14	15
<b>7</b>	7	8	9	10	11	12	13	14	15	16
<b>8</b>	8	9	10	11	12	13	14	15	16	17
<b>9</b>	9	10	11	12	13	14	15	16	17	18

Every student should have an addition table. Each time they master a set of facts, they should color in the addition facts they now know (or write them into the blank one). This lets them celebrate the facts they know and view the facts they have left to learn.

# Addition Table

<b>+</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>										
<b>1</b>										
<b>2</b>										
<b>3</b>										
<b>4</b>										
<b>5</b>										
<b>6</b>										
<b>7</b>										
<b>8</b>										
<b>9</b>										

## Twenty board

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

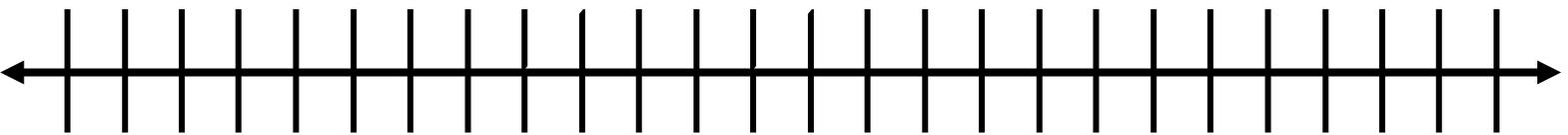
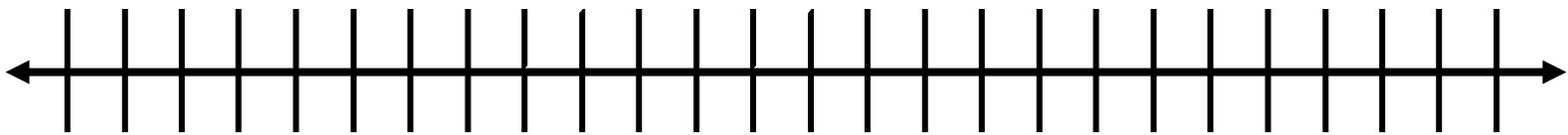
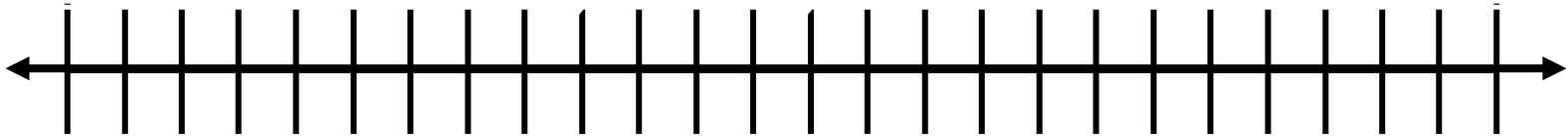
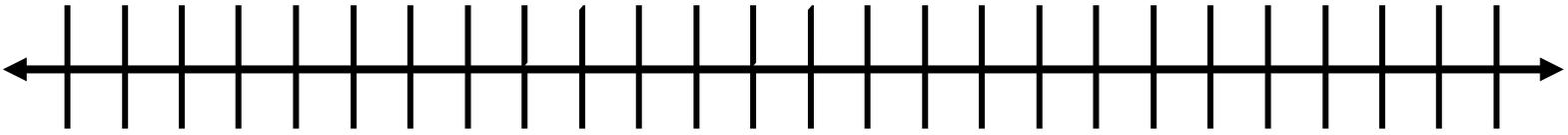
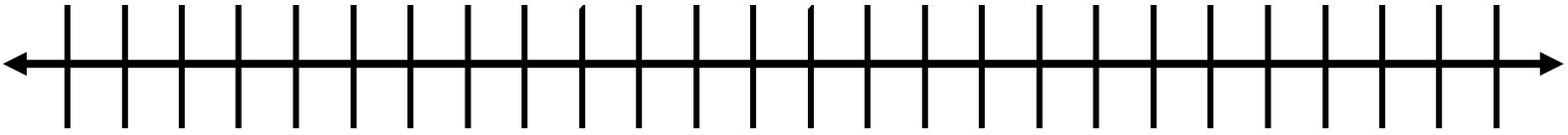
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

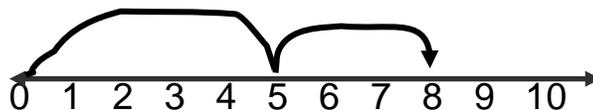
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

# Number line



Number lines are another tool students should have the opportunity to use when demonstrating their understanding of computation. Addition of positive numbers would be demonstrated with arrows going to the right. For example:  $5 + 3$ . This could look like:

Teacher note:  
adding negative  
numbers moves to  
the left.



# Foundational Numeration Skills

Students should not be made to work on their facts if they are still struggling with basic, foundational number sense skills. For those students that consistently count on their fingers or must always “count all” in order to find a correct answer to a simple addition problem, you will want to focus on increasing their skill of subitization. Subitizing must be mastered before students can be expected to master the fluency levels that are necessary with the basic facts.

## Two Types of Subitizing

(excerpt from *Subitizing: What is it? Why teach it?* by Doug Clements)

### Perceptual Subitizing

Perceptual subitizing is closest to the original definition of subitizing: recognizing a number without using other mathematical processes. For example, children might “see 3” without using any learned mathematical knowledge. Perceptual subitizing may involve mechanisms similar to those used by animals. Two-year-old children show this ability clearly (Gelman and Gallistel 1978).

Perceptual subitizing also plays an even *more* primitive role, one that most of us do not even think about because we take it for granted. This role is making units, or single “things,” to count. This ability seems obvious to us. However, “cutting out” pieces of experience, keeping them separate, and coordinating them with number words is no small task for young children. Even when they count their fingers, for example, they have to mentally “cut out” one part of the hand from the next to create units. They then have to connect each of these units with one, and only one, number word.

### Conceptual Subitizing

But how is it that people see an eight-dot domino and just know” the total number? They are using the second type of subitizing. Conceptual subitizing plays an advanced-organizing role. People who “just know” the domino’s number recognize the number pattern as a composite of parts and as a whole. They see each side of the domino as composed of four individual dots and as “one four.” They see the domino as composed of two groups of four and as “one eight.” These people are capable of viewing number and number patterns as units of units (Steffe and Cobb 1988).

Spatial patterns, such as those on dominoes, are just one kind. Other patterns are temporal and kinesthetic, including finger patterns, rhythmic, and spatial-auditory patterns. Creating and using these patterns through conceptual subitizing help children develop abstract number and arithmetic strategies (Steffe and Cobb 1988). For example, children use temporal patterns when counting on: “I knew there were three more, so I just said, ‘Nine . . . ten, eleven, twelve,’” rhythmically gesturing three times, one “beat” with each. They use finger patterns to figure out addition problems. Children who cannot subitize

conceptually are handicapped in learning such arithmetic processes. Children who can may subitize only small numbers at first. Such actions, however, can be stepping-stones to constructing more sophisticated procedures with larger numbers.

### **Conceptual Subitizing and Arithmetic**

The spatial arrangement of sets influences how difficult they are to subitize. Children usually find rectangular arrangements easiest, followed by linear, circular, and scrambled arrangements (Beckwith and Restle 1966; Wang, Resnick, and Boozer 1971). This progression holds true for students from the primary grades to college.

Finally, textbooks often present sets that discourage subitizing. Their pictures combine many inhibiting factors, including complex embedding, different units with poor form (e.g., birds that were not simple in design as opposed to squares), lack of symmetry, and irregular arrangements (Carper 1942; Dawson 1953). Such complexity hinders conceptual subitizing, increases errors, and encourages simple one-by-one counting.

We want to use conceptual subitizing to develop ideas about addition and subtraction. It provides an early basis for addition, as students "see the addends and the sum as in 'two olives and two olives make four olives' " (Fuson 1992, 248). A benefit of subitizing activities is that different arrangements suggest different views of that number.

Children can use familiar spatial patterns to develop conceptual subitizing of arithmetic. For example, students can use tens frames to visualize addition combinations (fig. 4). Such pattern recognition can assist students with mental handicaps and learning disabilities as they learn to recognize the five- and ten-frame configuration for each number. "These arrangements ... help a student first to recognize the number and use the model in calculating sums. It is this image of the number that stays with the student and becomes significant" (Flexer 1989).

Please contact the Elementary Mathematics Curriculum Department concerning any questions about the role of foundational numeration skills and lessons that can be utilized with your students. Resources are available electronically to assist in this work.

# Counting On

## One-more-than, Two-more-than

Students will be working on the facts that have addends of 1 or 2.

### Materials Needed:

- Connecting cubes
- White board
- Copy paper (1-2 each)
- One-, Two-More boards
- Dot cube and "4-9" cube

### Teaching Fact Activity—part 1

*Materials: connecting cubes, all-student response board or paper*

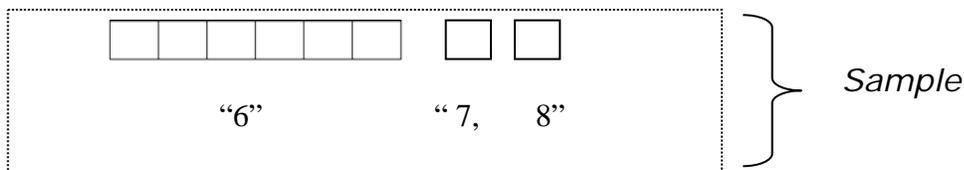
Let's take a look at a problem and talk about how we might solve it.

Tyrone has 3 pennies in his pocket. He found 1 penny on the sidewalk and picked it up. How many pennies does he now have? (*Students respond and have a few students explain their thinking. They may need to use pictures, manipulatives, number line, or symbols.*)

Let's try another one: Trisha has 6 pennies in her hand. Her aunt gives her 2 more. How many pennies does she now have? (*Students respond and a few explain their thinking using pictures, manipulatives, number line, or symbols.*)

We are going to work on an addition strategy that will help us with facts that have 1 or 2 as an addend. This is called the "Counting-On Strategy." We want to learn these facts so we know the answer by keeping track in our minds instead of on our fingers.

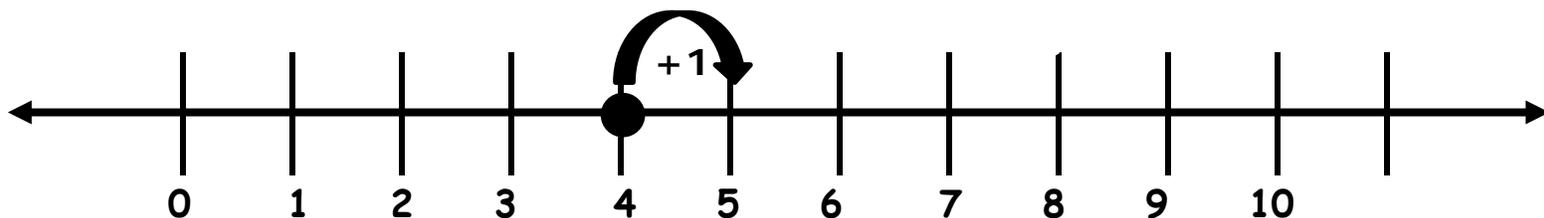
*Note: Begin teaching counting-on by using concrete objects, such as connecting cubes or color tiles. The student should be able to point to the greater addend, say "6", then count-on 1, "7, 8" (see picture):*



This will need to be modeled numerous times with students. Have students discuss how they know there are "6", then add on 2... The more students get the opportunity to explain their thinking, the stronger they will be with understanding the strategies. It is good to periodically stop and have students share with their shoulder partner or small group of students before discussing it as a whole group.

Sybilla Beckmann (math expert) suggests covering up the greater addend with a card or box so that the students must retain that number in their head before counting on with 1 or 2.

It is essential to then model this strategy with a number line (number line is a wonderful tool and is expected in the KCCRS). Start with the greatest addend and count-on 1 or 2 by jumping that number of spaces. Example:  $4 + 1 = 5$ .



It is also important that students have an understanding of the commutative property, called “turn around facts” by Everyday Mathematics, but use of the proper term as soon as possible is best. Using two different color of connecting cubes in one stick is an excellent picture for students. Demonstrate this with 5 green cubes and 2 red cubes [ $5 + 2$ ]. Then explain the commutative property and turn the stick of cubes around to show  $2 + 5$  [2 red cubes to the right of 5 green cubes]. Spend time allowing the students to discuss how they know the sum will be the same no matter which order the facts are added.

## Teaching Fact Activity—part 2

After children are comfortable using concrete objects you want to move them to ten frames as soon as possible. Use the Ten Frame Flashcards either under the document camera or in a PowerPoint.

1. You will “flash” the ten frame up for just a second or two and then take it away.
2. Students will mentally count on one more dot and then give the answer.
3. They can write their response on a white board or show you with a digit card. You want to make sure that all students are responding quickly.
4. Once you feel all students are comfortable with counting on one more, move to counting on two more.
5. Finally mix it up. Sometimes ask for 1 more and sometimes 2 more than what is shown on the ten frame.

## Teaching Fact Activity—part 3

After children are comfortable using concrete objects, ten frames, and the number line, it is time to switch to symbols for practicing counting-on. Here the children need to continue developing the Counting-On strategy by:

1. Identifying the larger addend as the number from which to count-on and retain that number in their head

2. Saying the number that comes after the larger addend
3. Counting-on to find the sum.

*For example, with the fact  $8 + 2$ , a student would think, "8...9, 10." Don't forget to remind them about the commutative property; so for  $2 + 8$ , a student would solve it the same.*

Let's look at how we could solve this problem:

Kelly has 2 pencils. Her friend gave her 6 more. How many pencils does Kelly have? *(Students respond and explain their thinking. How did they use the Counting-On strategy?)*

We are going to continue to practice the Counting-On strategy. *(Have the students explain how the connecting cubes and number line were used.)* Today we are going to continue to count on 1 or 2, but we will be doing the Counting-On in our heads.

*Give each student a "One More" board. Let's take our "One More" board to practice the +1 facts aloud together. (Put "One More" page under document camera.)* Notice the dot at the top of the page. We can use this dot as we add one to the numbers printed in the boxes. You will need to touch the number in the box, say its name and think about it in your head, then Count-On 1. You can touch the dot at the top of the page as you count-on 1 if that is helpful.

Let's do a few together. Start with "4". Touch it and say "FOUR", now Count-On one more and say "FIVE" (remind those that need to touch the dot at the top to do so if they need the physical touch at this stage). So, four and one more equal... *(five)*.

Next, touch and say "SIX"... *Count-On 1 more to make "SEVEN"*. Six plus one equals seven. *(May need to practice one more problem to model the process. Monitor to see that students are following directions.)*

Now, with your shoulder partners (or however you wish to pair students) complete the rest of the game board. *(Walk around & monitor student understanding.)*

*(Same procedure for the "Two More" boards.)*

## Practice Fact Activity

### **Show What You Know**

**Materials needed** *connecting cubes, copy paper*

I am now going to pass out paper and I want you to show me 5 examples of facts that use the Counting-On strategy. (*Show an example...you may choose the example pictured in the Teaching Fact Activity.*) You will need to draw the picture, and then write the numbers you would say. (*Students may trace around the cubes if necessary.*) Questions?

You may need to provide the students with a set of +1 and +2 cards to draw from to make their 5 examples.

## Practice Fact Activity

### **Counting On Dots**

**Materials needed:** *worksheet of "Counting-on Dice", ten-sided die, dot cube (blank cube that can be made to show 1 dot on three sides and 2 dots on the other three sides)*

1. Roll your ten-sided die and record the number in that square.
2. Find the sum for the rolled number and the dots next to the square. Write the sum in the circle provided.
3. Roll for the next problem and continue the rest of the page.
4. When all problems have been completed, turn the page over and show one of the number sentences on a number line.
5. Turn in your recording sheet.

Find a partner and practice your facts by rolling the ten-sided die and then Counting-On 1. After a few minutes, change to Counting-On 2.

## Practice Fact Activity

**One- or Two-More-Than Match** (*Teaching Student-Centered Mathematics* vol. 1, p.100)

**Materials needed:** *worksheet of "Counting On", pencil*

Students will start with a number, match that number with the one that is following the one-more-than or two-more-than rule, and then connect that with the corresponding basic fact.

## Practice Fact Activity

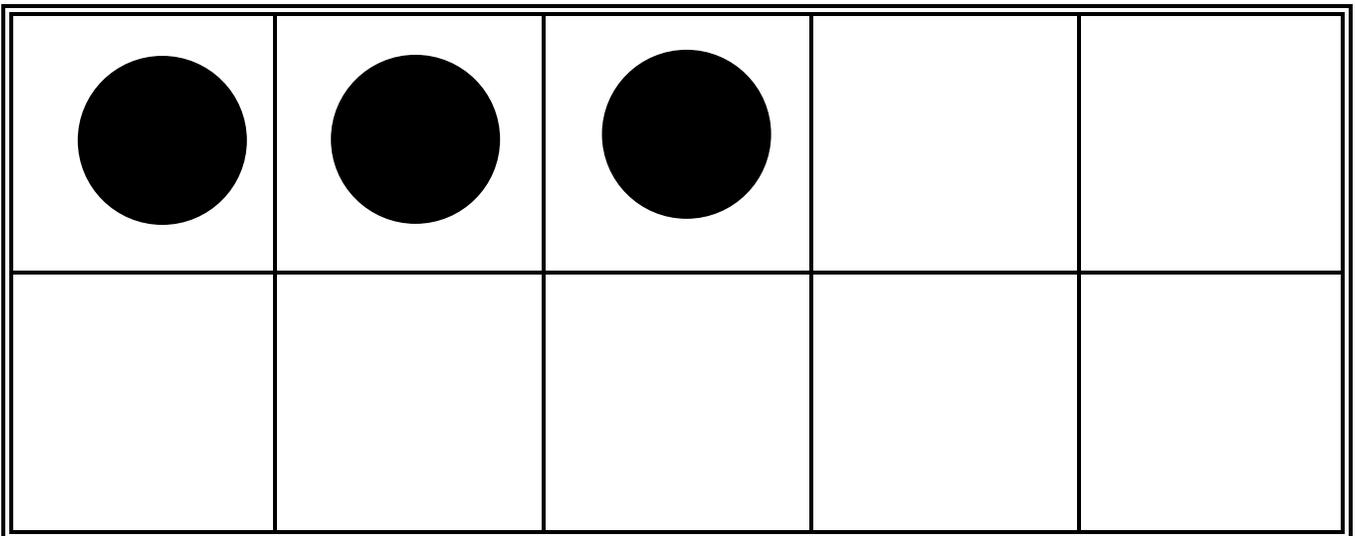
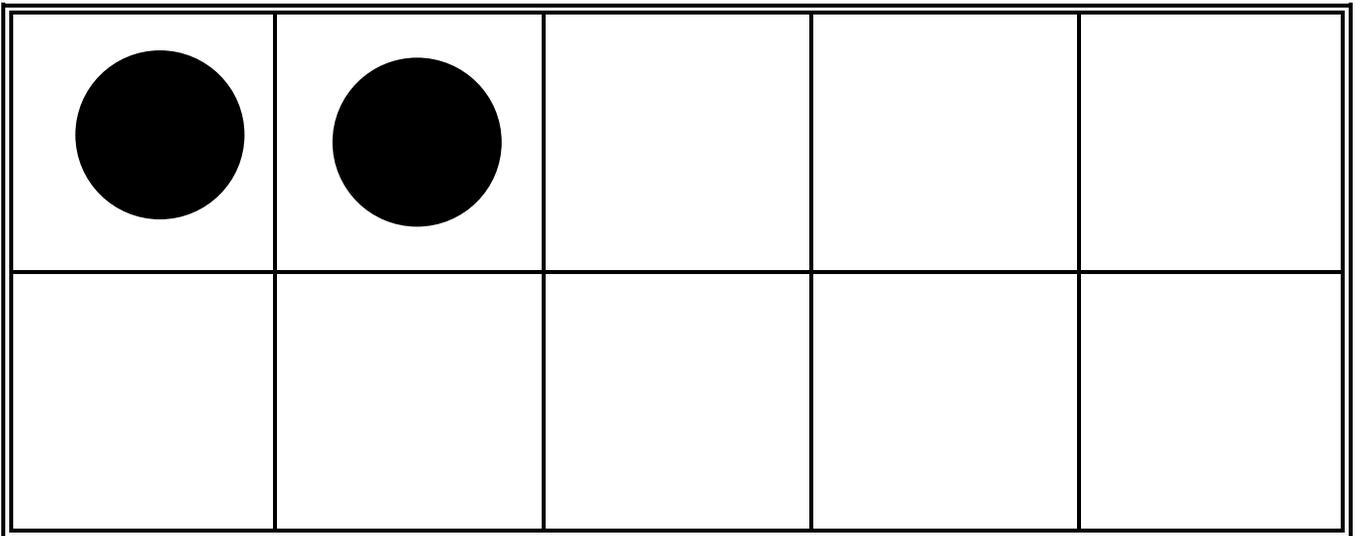
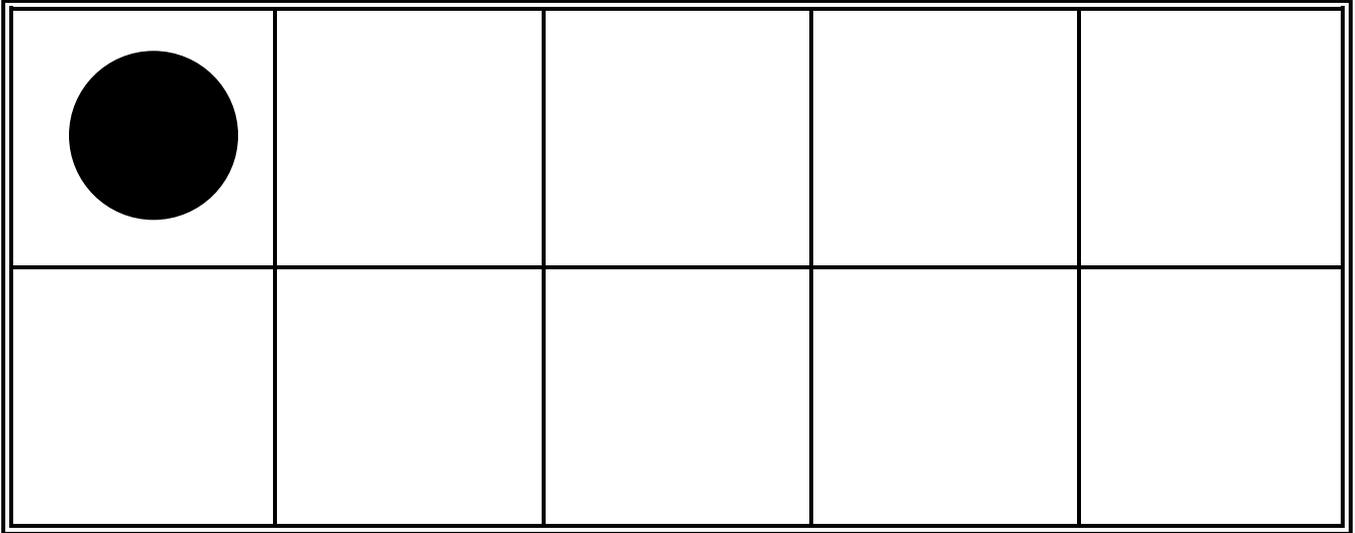
### **Random Number Counting On**

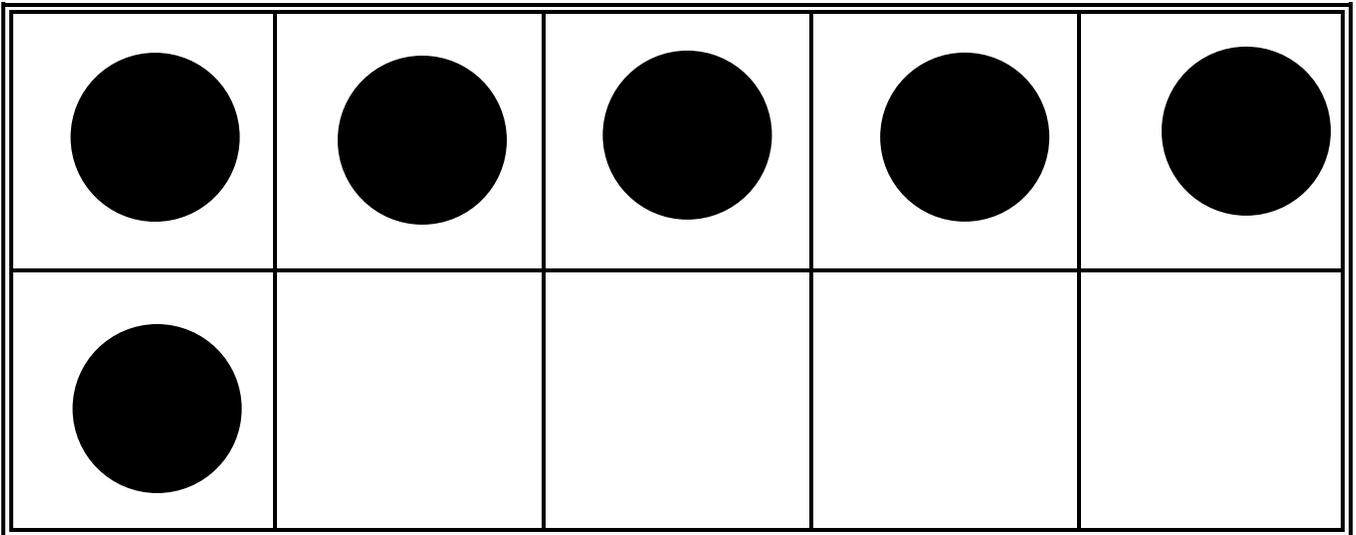
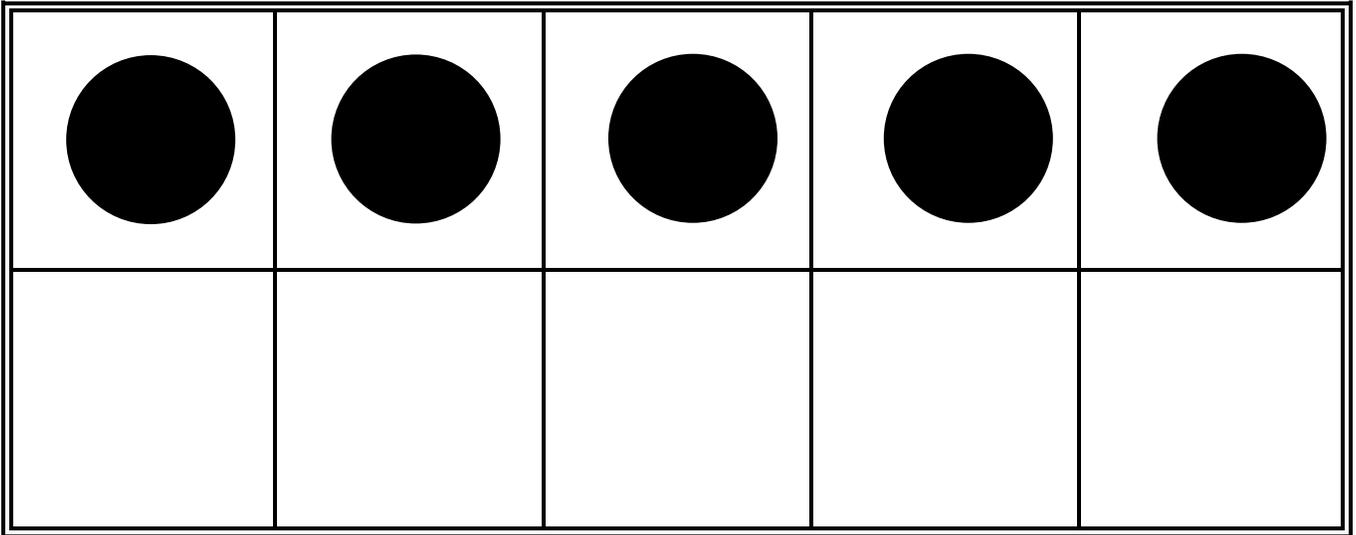
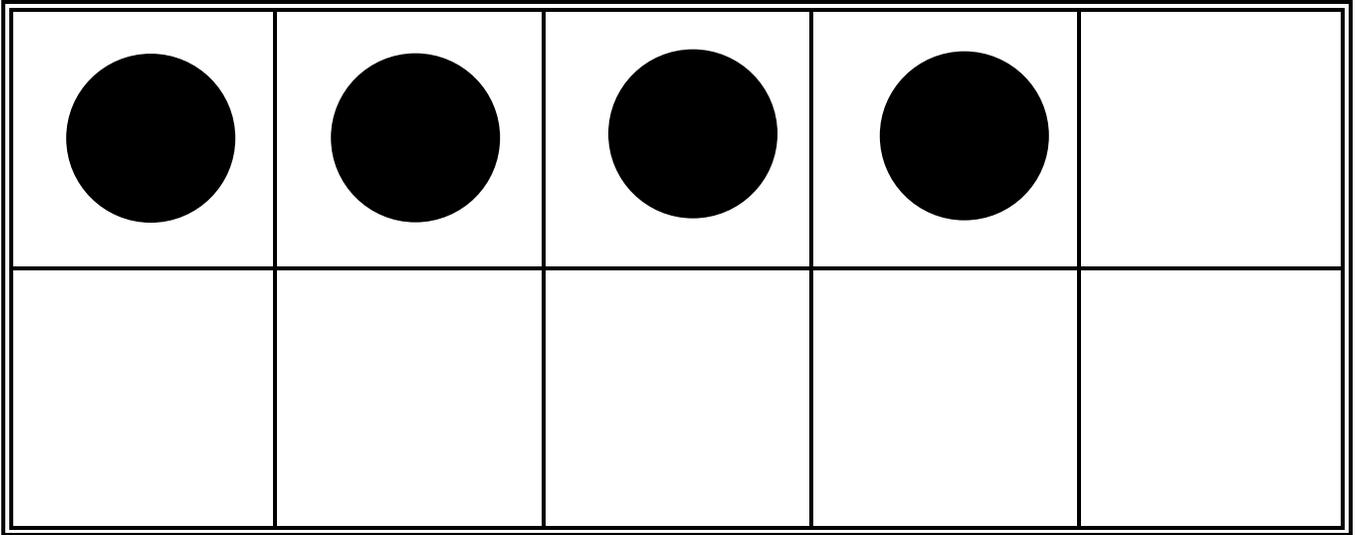
**Materials needed:** "Random Number ~ Counting-On" sheets, random number generator CD or spinner or 10-sided die

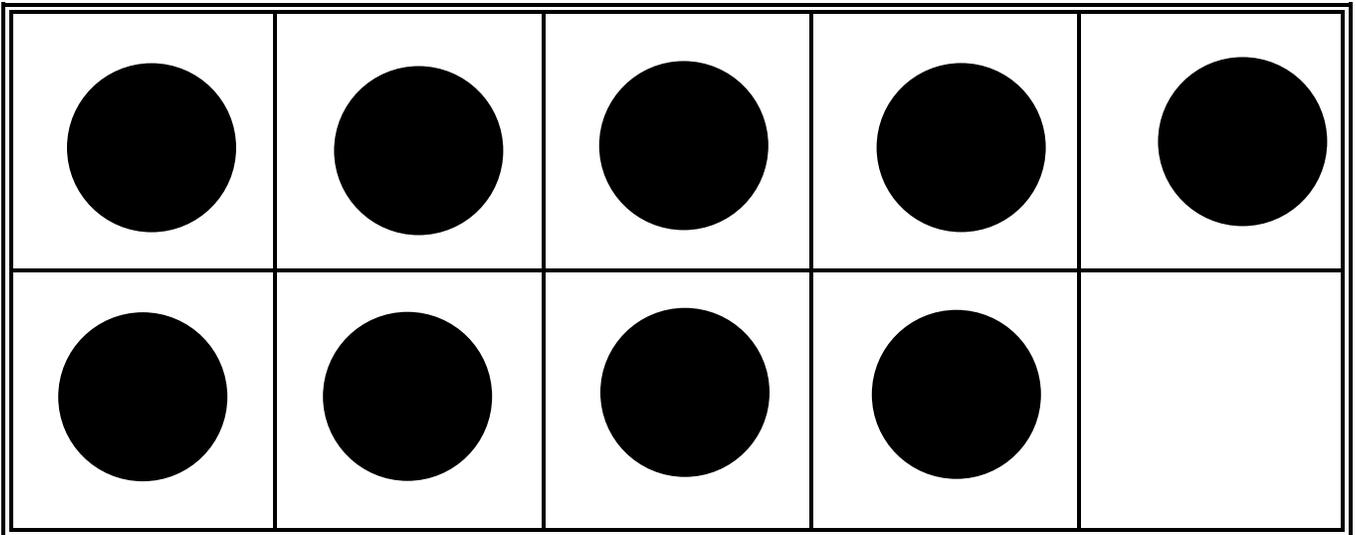
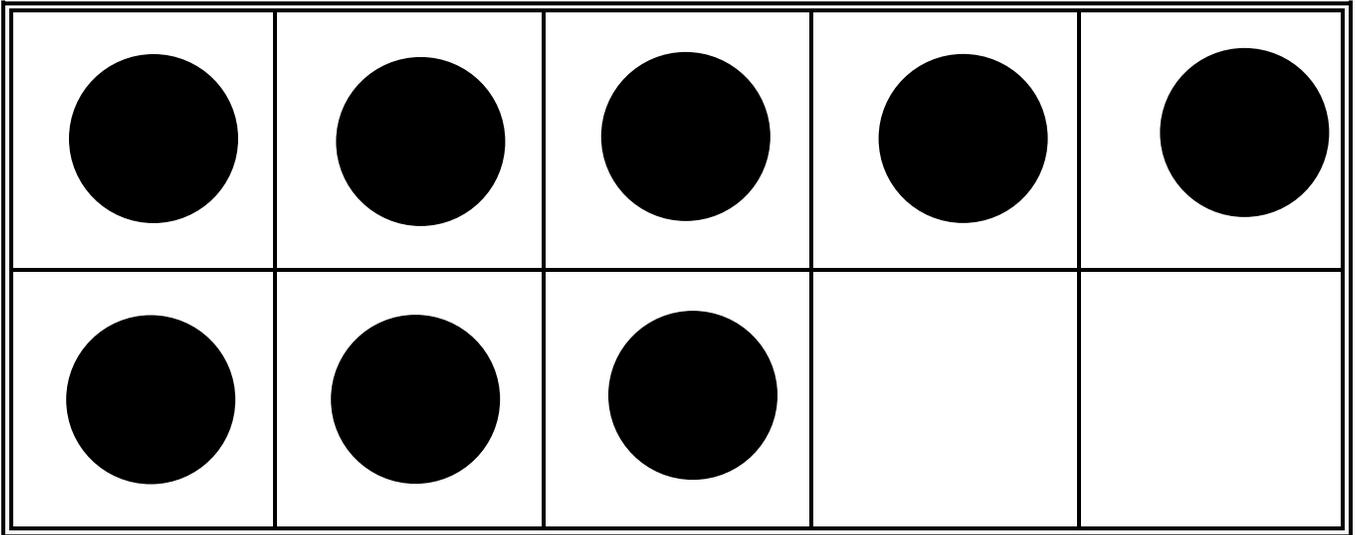
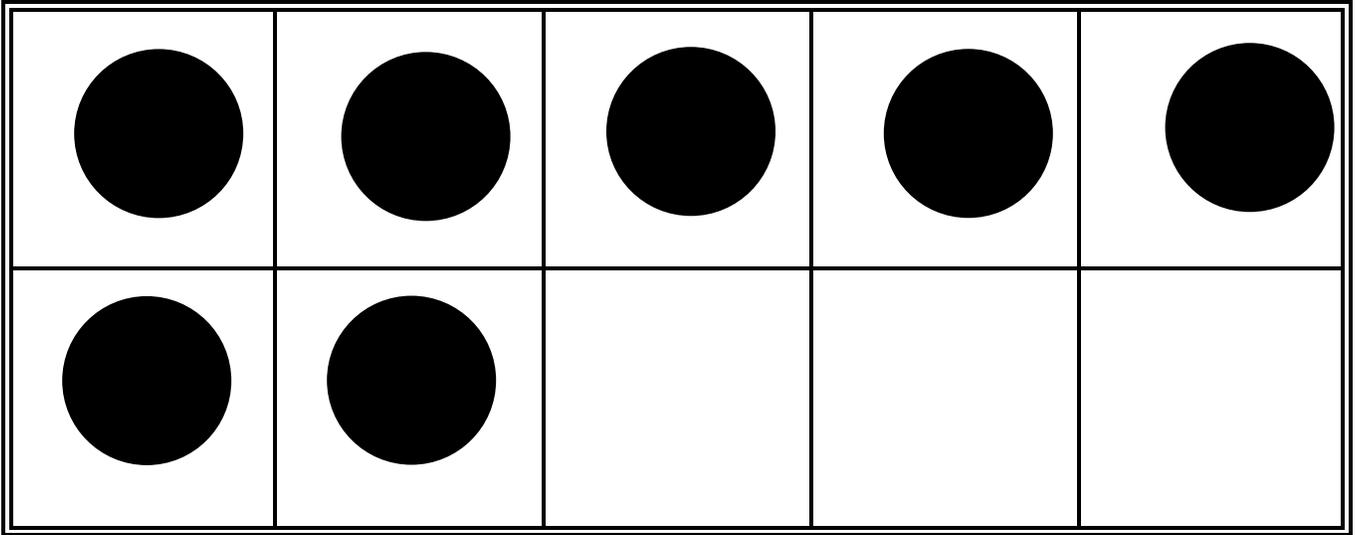
*The random number CD by Kim Sutton is a valuable tool for motivating students to practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information. Students also can't ask the CD to repeat itself!*

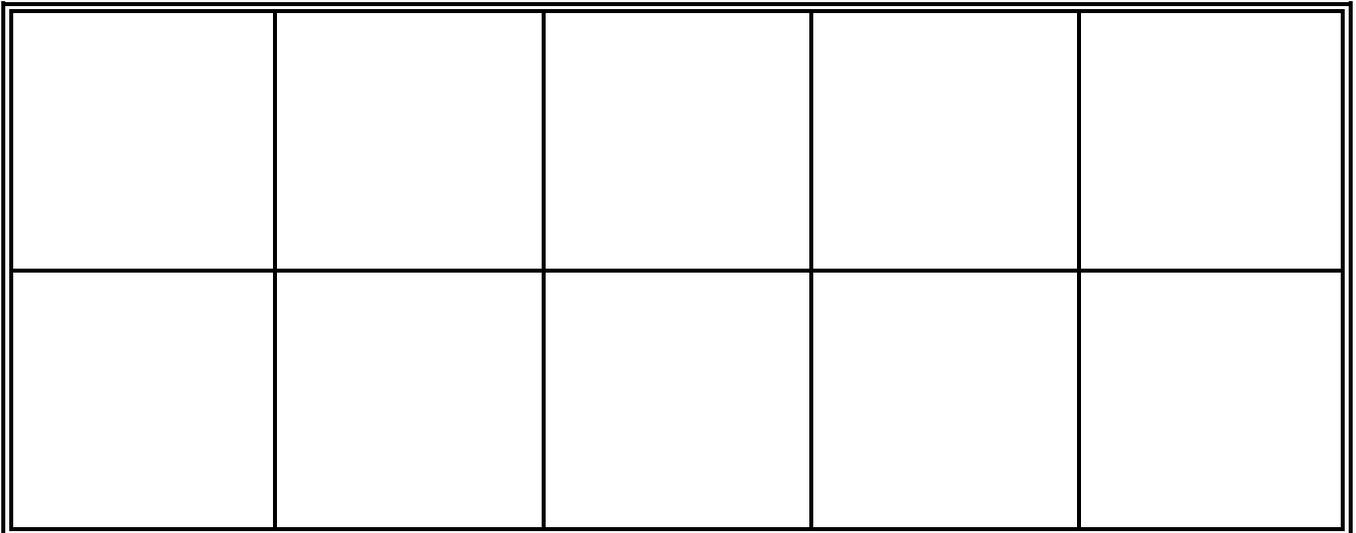
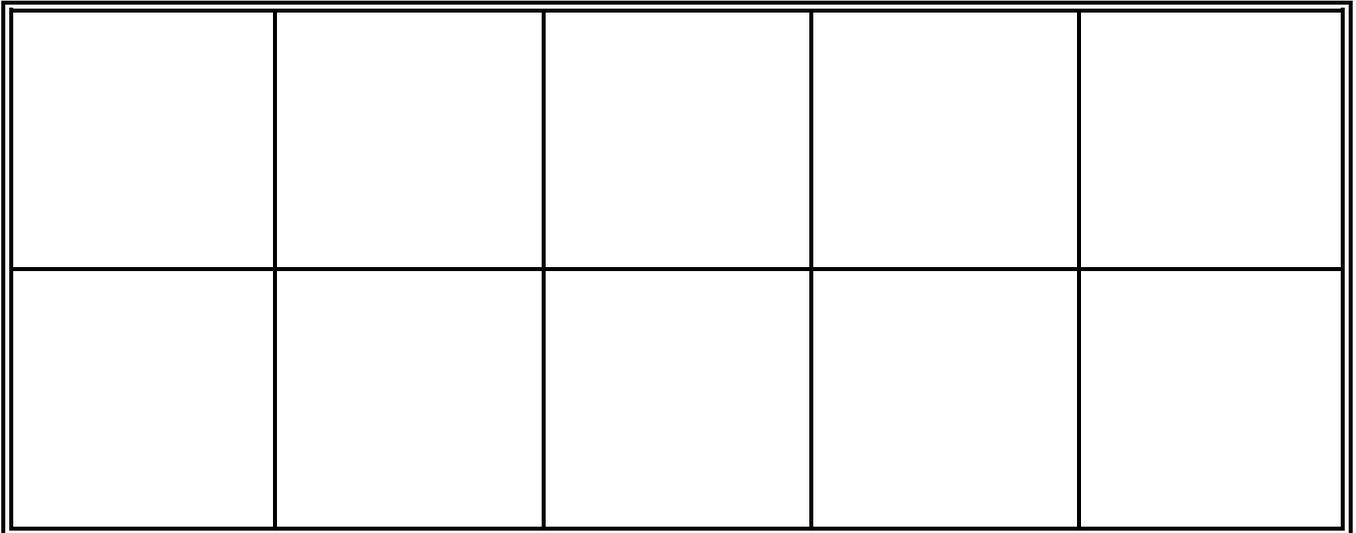
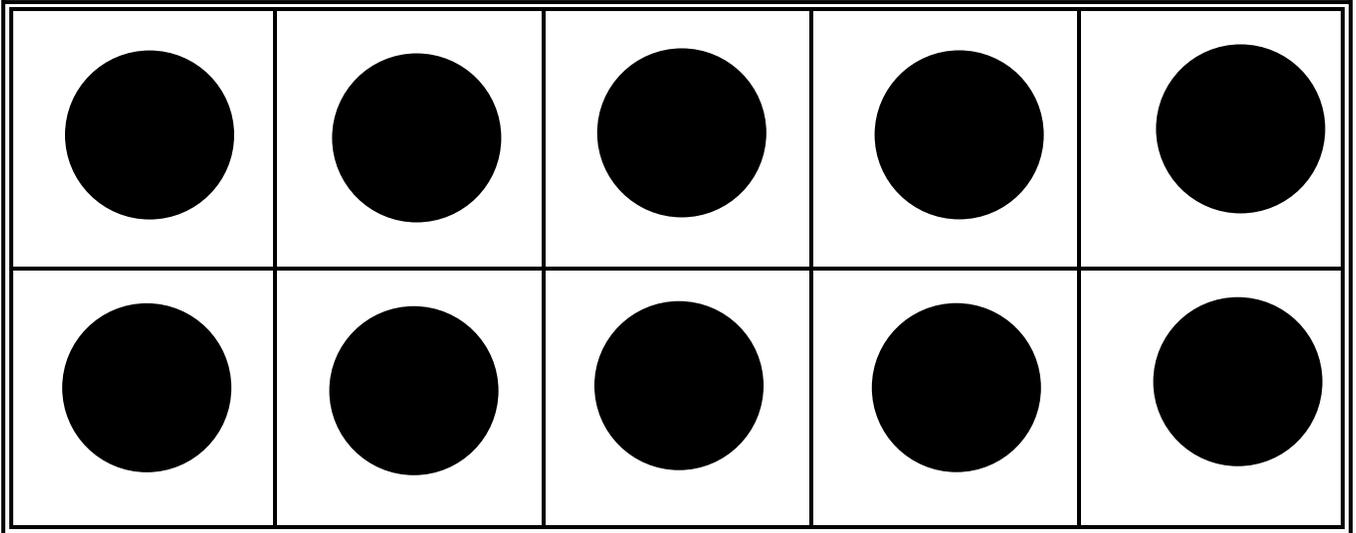
The random number CD can be used with any drill command using the four operations. When a number is called out, the student would perform that drill command (*add 1 or add 2*) and record the sum. This would continue down the columns.

# Ten Frame Flashcards









# One More ●

4

6

8

5

9

3

2

5

7

Two More



5	3	7
8	5	9
7	6	4

# Counting-On Dice

Materials: ten-sided die,  $\frac{1}{2}$  of this sheet for each student

	Count on ● ● = ○
	Count on ● = ○
	Count on ● ● = ○
	Count on ● ● = ○
	Count on ● ● = ○
	Count on ● = ○
	Count on ● ● = ○

	Count on ● ● = ○
	Count on ● = ○
	Count on ● ● = ○
	Count on ● ● = ○
	Count on ● = ○
	Count on ● ● = ○
	Count on ● ● = ○

# Counting-On

Start with a number on the right. Draw a line to the number in the middle that follows the one-more-than or two-more-than rule. Then, draw a line to its matching number sentence. One has been done for you.

## One-More-Than

## + 1 Fact

7	●	●	6	●	●	9 + 1
1	●	●	3	●	●	1 + 4
5	●	●	7	●	●	7 + 1
2	●	●	9	●	●	8 + 1
4	●	●	2	●	●	1 + 1
8	●	●	5	●	●	1 + 5
6	●	●	8	●	●	2 + 1
9	●	●	10	●	●	6 + 1

## Two-More-Than

## +2 Fact

9	●	●	9	●	●	7 + 2
5	●	●	5	●	●	2 + 1
2	●	●	3	●	●	9 + 2
8	●	●	6	●	●	4 + 2
7	●	●	7	●	●	2 + 5
3	●	●	10	●	●	6 + 2
6	●	●	4	●	●	2 + 3
4	●	●	11	●	●	2 + 2
1	●	●	8	●	●	8 + 2

# Random Number ~ Counting On

1.  +1 = \_\_\_\_\_

2.  +1 = \_\_\_\_\_

3.  +1 = \_\_\_\_\_

4.  +1 = \_\_\_\_\_

5.  +1 = \_\_\_\_\_

6.  +1 = \_\_\_\_\_

7.  +1 = \_\_\_\_\_

8.  +1 = \_\_\_\_\_

9.  +1 = \_\_\_\_\_

10.  +1 = \_\_\_\_\_

11.  +1 = \_\_\_\_\_

12.  +1 = \_\_\_\_\_

13.  +2 = \_\_\_\_\_

14.  +2 = \_\_\_\_\_

15.  +2 = \_\_\_\_\_

16.  +2 = \_\_\_\_\_

17.  +2 = \_\_\_\_\_

18.  +2 = \_\_\_\_\_

19.  +2 = \_\_\_\_\_

20.  +2 = \_\_\_\_\_

21.  +2 = \_\_\_\_\_

22.  +2 = \_\_\_\_\_

23.  +2 = \_\_\_\_\_

24.  +2 = \_\_\_\_\_

# Zero

## (Additive Identity)

Students will be working on the facts that have addends of 0.

### Materials Needed:

- Chalkboard/whiteboard
- counters
- part-part-whole/total mat

*There are 19 facts that have 0 as an addend. Though these facts are generally considered simple, there are students that over-generalize the idea that answers to addition problems must be bigger and get confused with the + 0 facts. Word problems involving zero are especially helpful for understanding zero facts. As you discuss the following problems, use concrete examples and drawings that show two parts with one part empty. Show students what the number fact looks like first and then show the abstract number sentence.*

### Teaching Fact Activity:

**Materials:** *paper or white boards, document camera or problems on chart paper*

We are going to use problems to help us understand adding zero. Let's talk about the following problems together.

There are 3 baby birds in a nest. No more birds have hatched.  
How many baby birds are in the nest?

» *Have students discuss the problem and illustrate it. It may also be helpful for some students to act it out.*

The next spring a robin built a nest. Rosa peeked into the nest one day and saw that it was empty. The next day she peeked and saw 4 eggs. How many eggs are in the nest?

» *Have students discuss the problem and illustrate it. It may also be helpful for some students to act it out.*

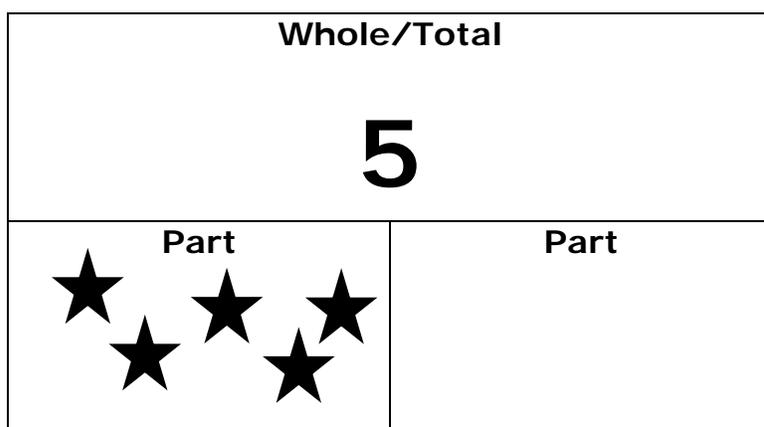
Both of these problems had 0 as an addend. Let's look a list of zero facts. *Make a list of about 10 zero facts (make sure the 0 is not always the same addend). Have students explain how these facts are alike and different*

## Practice Fact Activity

### Part-Part Whole-Zero Facts

**Materials needed:** *counters, part-part-whole mat (each), number cards or a ten-sided die*

Students will draw a card (or roll the die) and add zero. This will be recorded in each box at the bottom of the part-part whole mat both pictorially and its number sentence. The top of the mat allows the student to use manipulatives to solve each problem. For example:



$$5 + 0 = 5$$

# Part-Part-Whole/Total Mat

<b>Whole/Total</b>	
<b>Part</b>	<b>Part</b>

Draw how you solved each problem and write its number sentence.


# Doubles

Students will be working on the facts where both addends are the same.

## Materials Needed:

- Whiteboard or document camera
- Double Trouble poster
- Paper for "doubles booklet" (3 each)
- Doubles Partner Practice Page (Bristol page, one per pair of students)

*Children usually memorize doubles quickly and easily. Even so, the Doubles strategy is so critical in learning many of the other fact strategies that students need to have time to discuss this strategy with their peers in order to have a deep, foundational understanding.*

## Teaching Fact Activity:

**Materials:** paper, chalkboard/whiteboard

We are going to use problems to help us understand the doubles strategy. Let's talk about the following problems together.

Butterfly wings have the same pattern on both wings. If each wing has 5 dots, how many dots are on the butterfly?

» *Have students discuss the situation with their peers. Have them illustrate the problem. It may be helpful for some students to act it out.*

A car with four wheels is in the driveway. Another car just like it pulls into the driveway. How many wheels are on the driveway?

Vinnie got some new neon shoestrings. His shoe has 6 eyelet holes on each side. How many eyelet holes are there on one shoe?

## Practice Fact Activity:

### Double Trouble Book

**Materials needed** paper for "Double Trouble" booklet, crayons/markers, Doubles poster or bulletin board

*Because the Doubles facts are so important, students can create a "Double Trouble" booklet to use as a reference.*

1. *Take 3 sheets of paper and fold them in half.*
2. *Staple the fold and have students label each page with its own double themselves ( $0 + 0$ ,  $1 + 1$ , etc.)*
3. *Illustrate each of the double facts. Students may choose to draw the same pictures that are on display in your classroom, or they may choose their own double picture that makes sense to them.*

## Practice Fact Activity:

### Random Number ~ Doubles

**Materials needed:** "Random Number Doubles" sheets, random number generator CD or spinner or 10-sided die

*The random number CD by Kim Sutton is a valuable tool for motivating students with drill and practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information.*

The random number CD can be used with any drill command using the four operations. When a number is called out, the student would perform that drill command (*double the number heard*) and record the sum. This would continue down the columns.

# Double Trouble



$1 + 1 =$



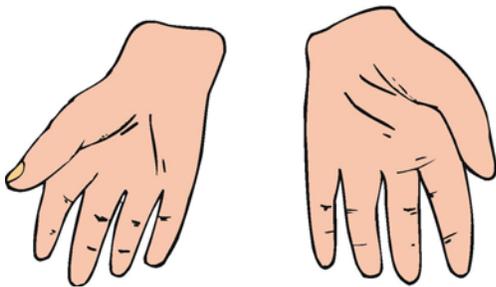
$2 + 2 =$



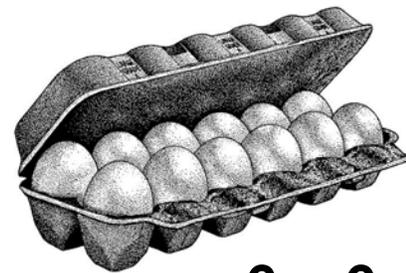
$3 + 3 =$



$4 + 4 =$



$5 + 5 =$



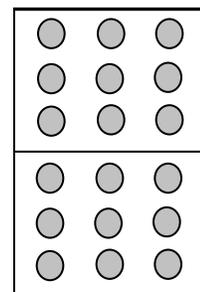
$6 + 6 =$

February						
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

$7 + 7 =$



$8 + 8 =$



$9 + 9 =$

# Random Number ~ Doubles

1. double  = \_\_\_\_\_

2. double  = \_\_\_\_\_

3. double  = \_\_\_\_\_

4. double  = \_\_\_\_\_

5. double  = \_\_\_\_\_

6. double  = \_\_\_\_\_

7. double  = \_\_\_\_\_

8. double  = \_\_\_\_\_

9. double  = \_\_\_\_\_

10. double  = \_\_\_\_\_

11. double  = \_\_\_\_\_

12. double  = \_\_\_\_\_

13. double  = \_\_\_\_\_

14. double  = \_\_\_\_\_

15. double  = \_\_\_\_\_

16. double  = \_\_\_\_\_

17. double  = \_\_\_\_\_

18. double  = \_\_\_\_\_

19. double  = \_\_\_\_\_

20. double  = \_\_\_\_\_

21. double  = \_\_\_\_\_

22. double  = \_\_\_\_\_

23. double  = \_\_\_\_\_

24. double  = \_\_\_\_\_

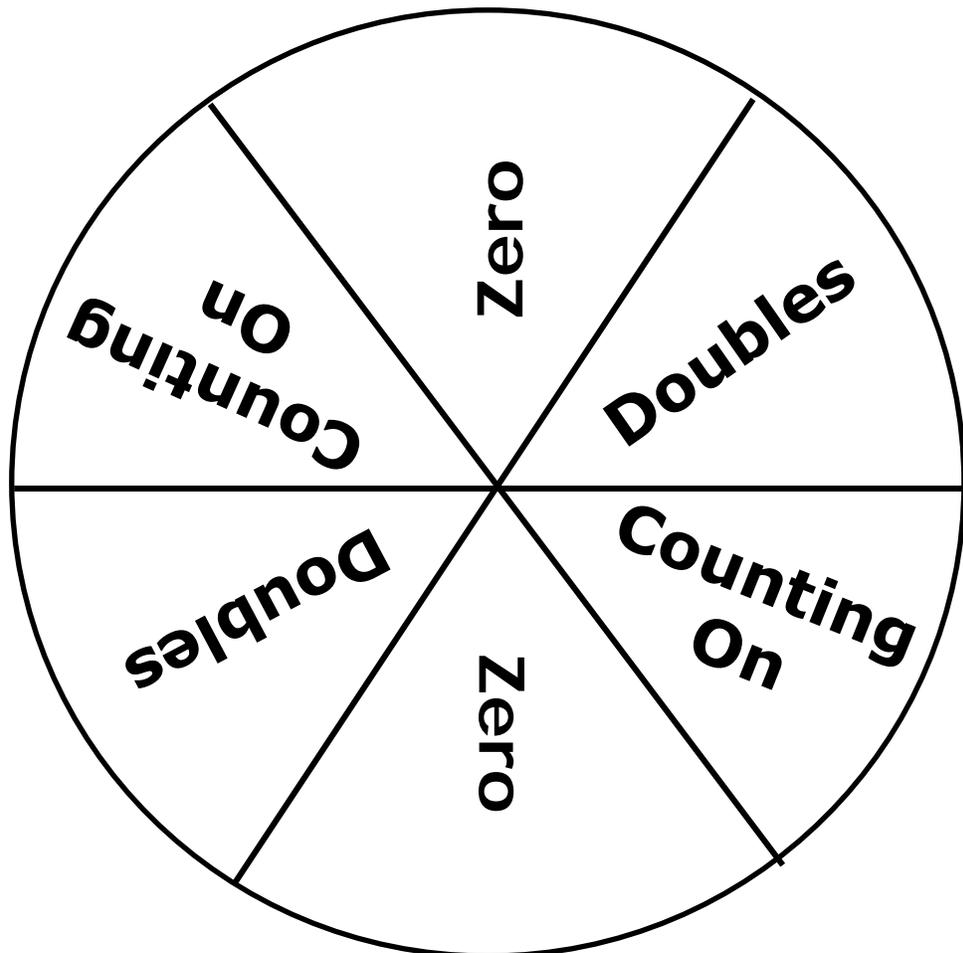
# Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "Counting On," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



# Strategy Focus Review

(Counting On, Zero, and Doubles)

$\begin{array}{r} 4 \\ + 4 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ + 8 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ + 4 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ + 2 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 0 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ + 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 8 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ + 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 0 \\ \hline \end{array}$
$\begin{array}{r} 1 \\ + 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ + 3 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ + 0 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ + 6 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ + 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ + 9 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ + 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ + 2 \\ \hline \end{array}$

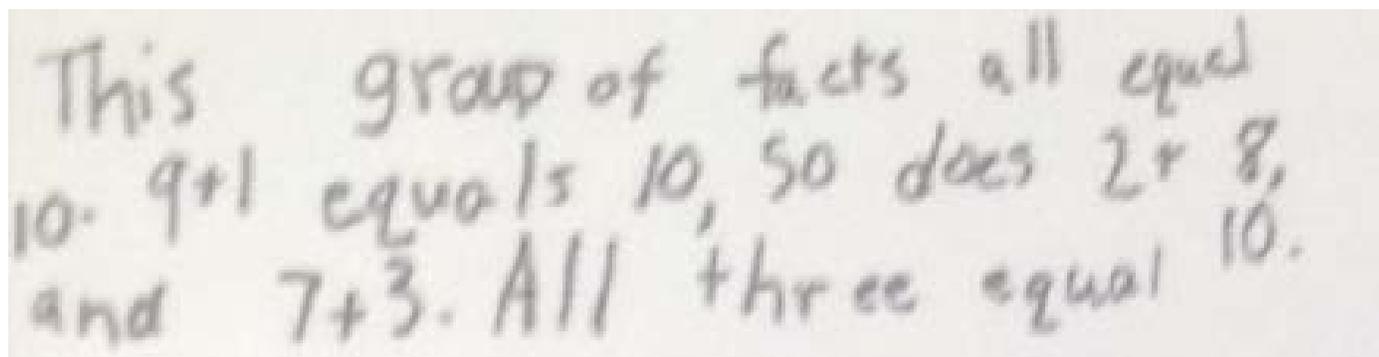
# Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 + 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts were chosen to be solved with that strategy.

Here is a student example of their reasoning.



This group of facts all equal 10.  $9+1$  equals 10, so does  $2+8$ , and  $7+3$ . All three equal 10.

# Strategy Sort

<b>Counting On</b>	<b>Zero</b>	<b>Doubles</b>
<b>Choose one strategy and explain why those facts are solved with the chosen strategy:</b>		

# Doubles + 1

Students will be working on the facts that have addends that are one away from each other.

## Materials Needed:

- "Double Helpers" worksheet
- Scissors
- Tape or glue
- Fact Cards
- Ten-sided die per pair

*Students will be working on a strategy that is a combination of the doubles and counting-on strategies. It involves facts whose addends differ by one. To understand these facts, students should compare them to the related double fact then complete the sum. For example, to solve the fact  $4 + 5$ , a student would think, " $4 + 4 = 8$ , so  $4 + 5$  is one more which would be a total of 9."*

## Teaching Fact Activity

**Materials:** paper or white board, document camera

We are going to use doubles to help us understand the doubles +1 strategy. Let's talk about the following problems together.

Today is the 7<sup>th</sup> day of the month? What will the date be in 8 days?

» *Have students discuss how to solve the problem with a Doubles + 1 strategy. Let them discuss it with each other and with the whole group. Have students illustrate the problem. Some students may find it helpful to act it out.*

After a party, your mom gathered up the leftover cans of pop. She found a pack of 6 in the kitchen and then 7 more cans in the refrigerator. How many cans of pop did she find?

There are two cars in the parking lot. One is a convertible and the other is an SUV with an extra wheel attached to the back. How many wheels are there in the parking lot?

I have some problems I want you to examine

*Put these problems under the document camera or on chart paper:*

$6 + 7 =$	$5 + 4 =$	$4 + 3 =$	$5 + 6 =$	$7 + 8 =$
9	6	3	8	7
<u>+8</u>	<u>+5</u>	<u>+4</u>	<u>+9</u>	<u>+6</u>

As each problem is revealed or written on chart paper, have students discuss which double fact is close to the problem shown. *(Some may double the smaller number and add one while others may double the larger and subtract one.)* Discussion and explanation of how the double is "hidden" in each of these problems is an important component of truly understanding the Doubles +1 strategy.

## Teaching Fact Activity

**Materials:** *fact cards, Doubles page*

*(Pass out the fact cards copied from the next pages to your students and place the Doubles page under the document camera. You may want to have students work as partners.)*

Look at the fact that has been passed out to you. Think about the doubles fact that helps you solve this problem. *(If students respond that they "just know it", have them approach the problem as though they don't know it.)*

Be ready to explain your thinking. *(If students are working with a partner, they will need to spend time discussing their thoughts. After students have had time to process their thoughts, have them share which double helps them solve their problem and how it helped.)*

## Practice Fact Activity

### Doubles Helpers

**Materials needed:** *"Doubles Helpers" worksheet, scissors, glue/tape*

You will need scissors, a pencil, and tape or glue for this next practice activity. At the bottom of your worksheet, there are six facts that need to be cut out. Decide which double fact could help you solve the fact that has been cut out. Tape or glue it to your paper and show or explain how that double fact helped.

## Practice Fact Activity

**Materials needed:** ten-sided die per pair, paper and pencil

*Give each pair of students the supplies from above.*

You and your partner are going to take turns rolling your die. The first person will roll and complete the doubles-plus-one fact. For example, when you roll a 7, you will need to say, "Seven plus eight *(because 8 is one more than 7)* is fifteen." The partner will take the paper and write the fact down with the answer. Then you will switch roles.

## Practice Fact Activity:

### **Random Number ~ Doubles + 1**

**Materials needed:** "Random Number Doubles" sheets, random number generator CD/spinner/10-sided die

*The random number CD by Kim Sutton is a valuable tool for motivating students with drill and practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information.*

The random number CD can be used with any drill command using the four operations. When a number is called out, the student would perform that drill command (*double the number heard and add 1*) and record the sum. This would continue down the columns.

# Doubles

$$\begin{array}{r} 6 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +8 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +9 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +2 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$$

# Fact Cards

$$\begin{array}{r} 7 \\ +8 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ +8 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ +5 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +9 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$$

# Double Helpers



Cut out the number facts at the bottom of the page. Tape or glue the "Double-Plus-One" fact beside the double fact that helps. Show how it helped you solve the cut out fact.

$\begin{array}{r} 6 \\ +6 \\ \hline \end{array}$		
$\begin{array}{r} 4 \\ +4 \\ \hline \end{array}$		
$\begin{array}{r} 7 \\ +7 \\ \hline \end{array}$		
$\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$		
$\begin{array}{r} 3 \\ +3 \\ \hline \end{array}$		
$\begin{array}{r} 8 \\ +8 \\ \hline \end{array}$		

Cut out facts below:

$5 + 6 =$	$4 + 5 =$	$4 + 3 =$
$8 + 7 =$	$9 + 8 =$	$6 + 7 =$

# Random Number ~ Doubles + 1

1. double  +1 = \_\_\_\_\_

2. double  +1 = \_\_\_\_\_

3. double  +1 = \_\_\_\_\_

4. double  +1 = \_\_\_\_\_

5. double  +1 = \_\_\_\_\_

6. double  +1 = \_\_\_\_\_

7. double  +1 = \_\_\_\_\_

8. double  +1 = \_\_\_\_\_

9. double  +1 = \_\_\_\_\_

10. double  +1 = \_\_\_\_\_

11. double  +1 = \_\_\_\_\_

12. double  +1 = \_\_\_\_\_

13. double  +1 = \_\_\_\_\_

14. double  +1 = \_\_\_\_\_

15. double  +1 = \_\_\_\_\_

16. double  +1 = \_\_\_\_\_

17. double  +1 = \_\_\_\_\_

18. double  +1 = \_\_\_\_\_

19. double  +1 = \_\_\_\_\_

20. double  +1 = \_\_\_\_\_

21. double  +1 = \_\_\_\_\_

22. double  +1 = \_\_\_\_\_

23. double  +1 = \_\_\_\_\_

24. double  +1 = \_\_\_\_\_

# Combinations of Ten

Students will be working on the facts that have sums of ten.

## Materials Needed:

- Ten-frame flash cards (pair)
- Deck of cards (pair)
- Blank ten-frame (optional)

*The ten-frame model is valuable in seeing essential number relationships. The ten-frame is one model that assists children in learning combinations that make 10.*

## Teaching Fact Activity

**Materials:** *paper or white board, document camera, blank Ten Frames, counters*

We are going to use problems to help us understand the “Combinations of Ten” strategy. Let’s talk about the following problems together.

Linda is supposed to have 10 crayons. She has 7 in her box. How many more does she need to have all of her crayons?

» *Have students discuss the Combinations of Ten strategy. Show a blank ten frame and use counters to show how this tool can be used to solve the problem. Pass these tools out for the students to use.*

Celia has 3 of her crayons. How many does she need to make 10?

David has 4 crayons in his desk and 6 crayons on the floor. How many crayons does he have?

Students may need more work with the Ten Frame if this is not a tool that is familiar to them. Give more situations and have them use the tools to find the combinations. Eventually, students should start to see a pattern and you should encourage them to discuss this and have the students to write down all combinations of ten.

## Teaching Fact Activity:

**Ten Frame Flash Cards** ([Teaching Student-Centered Mathematics \[First Edition\]](#); Activity 2.15, pg. 47)

**Materials:** *Ten Frame flash cards (use the set from the Counting On strategy)*

We are going to continue the work with our ten facts by using the Ten Frame flashcards. To start, let’s warm up with saying how many dots you see on each card. *(Flash each card, in random order, and have students state the number of dots that they see. Stop periodically and have students explain how they know they have the correct answer.)*

Now let's go through the cards again, but this time you will tell me how many spaces you see on each card. *(Flash each card and have students state the number of spaces they see. Stop periodically and have students explain how they know they have the correct answer.)*

Now let's go through the cards again, and this time around you will tell me a "ten-fact". *(For a card with 7 dots, the response is "seven and three is ten.")*

These activities can be used in independent or small groups.

## Practice Fact Activity

### Catch a Ten

**Materials:** deck of cards

**Directions:**

1. Two or more players may play this game.
2. The player with the most buttons or the most pockets gets to shuffle the deck of cards first and be the dealer.
3. The dealer deals out the cards (with face cards removed) to every player until all of the cards have been dealt.
4. Every player places their cards face down in front of them.
5. When the dealer says "Go" every player turns the top card over so that all players can see it. All players look at the cards and try to see if they can make a sum of 10 with two or more of the cards shown.
6. When a player sees a sum of 10, they must "catch" the cards by slapping them with their hands and saying the addition sentence. (Example – a player sees a 6 and a 4, so the cards are slapped to "catch" them and the number sentence is said – "6 plus 4 equals 10")
7. If everyone agrees that this number sentence is correct, the player takes the cards and puts in a pile that is separate from their original deck. These "caught" cards will be points.
8. If players do not see a 10, the dealer will say "Go" and the next card in the stack will be turned over and placed on top of the other card that was shown. If a 10 is made this time, the cards below must stay. The "catcher" can only take the cards that make a sum of 10. If a sum of 10 can be made once those cards are gone, then they can be "caught" as well.
9. Play continues in this way until all cards have been turned over and the winner is the one with the largest number of "caught" cards.

## Practice Fact Activity:

### Tens Go Fish

Materials: deck of cards—remove face cards (small group/pair)

#### Directions:

*The object of the game is to get two cards that total 10.*

1. Each player is dealt five cards. The rest of the cards are placed face down in the center of the table.
2. If you have any pairs of cards that total 10, put them down in front of you and replace those cards with cards from the deck.
3. Take turns. On a turn, ask one other player for a card that will go with a card in your hand to make 10.
4. If you get a card that makes 10, put the cards down. Take one card from the deck. Your turn is over.  
If you do not get a card that makes 10, take the top card from the deck. Your turn is over.  
If the card you take from the deck makes 10 with a card in your hand, put the pair down and take another card.
5. If there are no cards left in your hand but still cards in the deck, you take two cards.
6. The game is over when there are no more cards.
7. At the end of the game, make a list of the number pairs you made.

# Ten Frame





# Catch a Ten!

## Materials:

- Deck of cards with face cards taken out and Aces represent the number 1

## Directions:

1. Two or more players may play this game.
2. The player with the most buttons or the most pockets gets to shuffle the deck of cards first and be the dealer.
3. The dealer deals out the cards (with face cards removed) to every player until all of the cards have been dealt.
4. Every player places their cards face down in front of them.
5. When the dealer says "Go" every player turns the top card over so that all players can see it. All players look at the cards and try to see if they can make a sum of 10 with two or more of the cards shown.
6. When a player sees a sum of 10, they must "catch" the cards by slapping them with their hands and saying the addition sentence. (Example – a player sees a 6 and a 4, so the cards are slapped to "catch" them and the number sentence is said – "6 plus 4 equals 10")
7. If everyone agrees that this number sentence is correct, the player takes the cards and puts in a pile that is separate from their original deck. These "caught" cards will be points.
8. If players do not see a 10, the dealer will say "Go" and the next card in the stack will be turned over and placed on top of the other card that was shown. If a 10 is made this time, the cards below must stay. The "catcher" can only take the cards that make a sum of 10. If a sum of 10 can be made once those cards are gone, then they can be "caught" as well.
9. Play continues in this way until all cards have been turned over and the winner is the one with the largest number of "caught" cards.

**Another option: Once your child is proficient in "catching" 10s, you may want them to "catch" another sum, such as 12, 9, 20, etc.**



# Tens Go Fish



Materials: deck of cards—remove face cards (per pair)

## Directions:

The object of the game is to get two cards that total 10.

1. Each player is dealt five cards. The rest of the cards are placed face down in the center of the table.
2. If you have any pairs of cards that total 10, put them down in front of you and replace those cards with cards from the deck.
3. Take turns. On a turn, ask one other player for a card that will go with a card in your hand to make 10.
4. If you get a card that makes 10, put the cards down. Take one card from the deck. Your turn is over.  
If you do not get a card that makes 10, take the top card from the deck. Your turn is over.  
If the card you take from the deck makes 10 with a card in your hand, put the pair down and take another card.
5. If there are no cards left in your hand but still cards in the deck, you take two cards.
6. The game is over when there are no more cards.
7. At the end of the game, make a list of the number pairs you made.



# Make a Ten

Students will be working on the facts that have sums greater than ten.

## Materials Needed:

- Student copies of the Ten Frame
- Counters for the Ten Frame (or beans, unit cubes, connecting cubes, etc.)

This strategy can be used with facts that have at least one addend of 8 or 9 (sometimes 7). Students build onto the 8 or 9 up to 10 and then add on the rest. For example, with  $6 + 8$ , start with 8, then 2 more makes 10, and that leaves 4 more for 14.

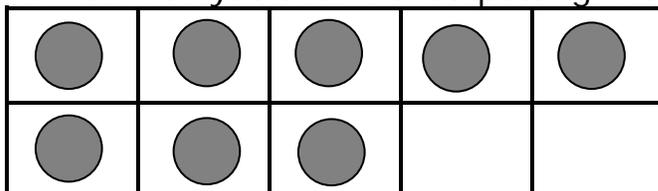
## Teaching Fact Activity:

**Materials:** Ten Frame, connecting cubes/counters

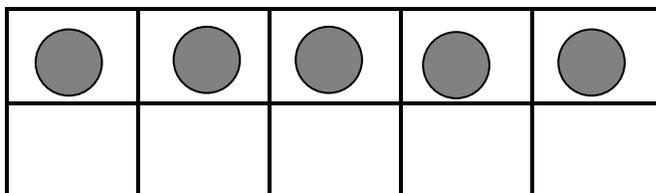
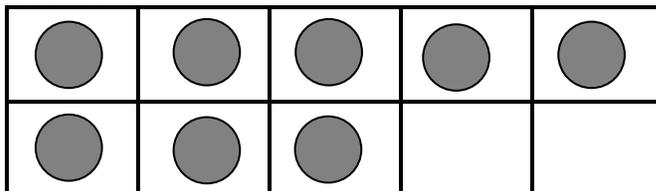
### Directions:

1. Have students quickly give you the answer to  $10 + 4 = \underline{\quad}$ . "What about  $10 + 8 = ?$ " "What about  $10 + 7 = ?$ " Ask students why these problems are easy to add so quickly?
2. Have students explain their thinking and reinforce their ideas. Explain that we will be learning a strategy that will help them make their addition fact problems with sums greater than 10 into facts of " $10 + \underline{\quad}$ ".
3. Write " $8 + 5 = \underline{\quad}$ " on the chalkboard or chart paper, then put the Ten Frame under the document camera with your counters close by for the demonstration.
4. Ask students why they think this is called a ten-frame. Students will tell you it is because there are 10 squares in the rectangle. They may say that the rectangle acts like a "frame" for the 10 squares.
5. Direct their attention back to the problem and ask the students which number is the larger addend. (8) This number will be placed into the ten frame, but the counters have to be placed into the frame like you are reading - starting at the top to the left and work your way to the right before you go down to the next row. Explain that they have to be placed in this way so that everyone is getting the same picture as we fill the ten-frame. This must be consistent! This is making an imprint in their brains.
6. Demonstrate with the counters. Make sure that you very purposefully place the counter into each square starting at the top and moving to the right.

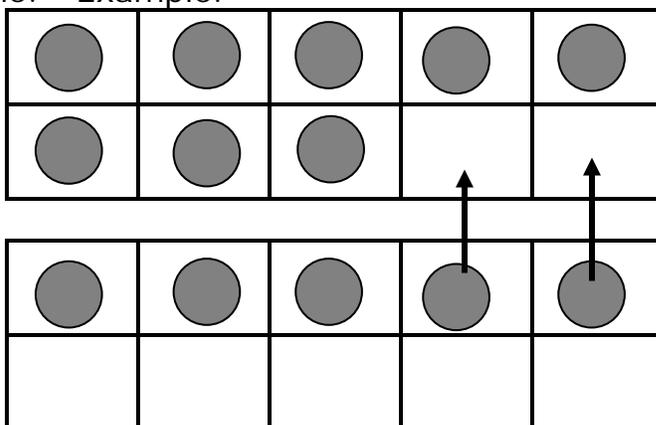
Example of the frame when you are finished putting in the 8 counters:



7. Have students tell you which addend is left to use on the ten-frame. (5) These 5 do not go into the frame. They are placed underneath in another empty 10-frame. The ten-frame page will look like the picture below when you are done setting up the problem:



8. Ask students how many spaces are empty in our ten-frame. (2) "Let's move up 2 counters from the 5 counters below the ten-frame to fill it up." "Did I add any counters from my bag?" "No, I just moved 2 from the 5 I already had to fill the ten-frame." Example:



9. "Now I have a  $10 + \underline{\quad} =$  problem. What is it?" ( $10 + 3 = 13$ ) "If  $10 + 3 = 13$ , and all I did was move up 2 from the 5 I already had below my ten-frame (move the 2 counters back down and then back up into the top ten frame), then I know that  $8 + 5 =$  to what?" (13) I just made it into a  $10 + \underline{\quad} =$  problem. I just grouped what I had to make a 10 and then added the rest to get my answer."
10. Model with another problem, but put the larger addend as the second number in your problem. Example:  $6 + 9 = \underline{\quad}$ . Emphasize that the greater number will go into the ten-frame so that you can fill it up faster. So the 9 will go into the ten frame and the 6 will go down below in the other 10 frame.
11. Pass out the student copies of the ten-frame along with counters (or beans, or connecting cubes, or unit cubes...) and have the students work on the next few problems with you. Circulate the room after each step to make sure that all of the students are correctly placing their counters. You are trying to make it so that the students will start getting a mental picture of the ten-frame in

their heads. Eventually they will be able to maneuver the counters in their heads quickly. ( $6 + 9$ ;  $5 + 9$ ;  $8 + 5$ ;  $8 + 6$ ...)

*(After a few days of practicing with the counters and moving them up into the ten frame to get a group of 10, tell students they can no longer move the counters from the bottom of the ten frame up into the frame to fill it. They must do it in their heads. They need to mentally move the counters from the bottom up into the ten-frame in their heads, but not their hands. (Some students will have already been at this step before other students.) After a few days of mentally moving the counters, the students cannot even put the smaller addend under the ten-frame at all. They must mentally place the counters underneath and then mentally move them up into the ten-frame. For the last step students don't put any counters into the ten-frame at all. They are to mentally place counters into the ten-frame and follow through with the rest of the procedure. Some students may still need to have the ten-frame on hand to look at while they are mentally moving the counters, but others will be able to do this quickly in their heads.)*

Eventually, you should have the students use the hundred chart. They should see the pattern of moving down one row when adding 10 to any number. This strategy will be useful when they get to the +9 strategy later.

# Ten Frame



# Doubles + 2

Students will be working on the facts that have addends that are two apart from each other.

## Materials Needed:

- Random Number ~ Doubles +2 worksheet
- Random number generator cd/dice/spinner

*Of the handful of remaining facts, three have addends that differ by 2:  $3 + 5$ ,  $4 + 6$ ,  $5 + 7$ . Some students find it easy to extend the idea of the near-doubles to double plus 2. For example,  $3 + 5$  is double 3 and 2 more.*

## Teaching Fact Activity

**Materials:** paper, chalkboard/whiteboard/pencil

We are going to use problems to help us understand the doubles +2 strategy. Let's talk about the following problems together.

Larry has 5 counters in one hand and 7 in the other. How many counters does he have?

*Have students discuss and illustrate the problem. It may also be helpful to act it out.*

Mom has 8 eggs in the refrigerator and bought half a dozen more. How many eggs does she now have?

## Practice Fact Activity:

Random Number Double +2

**Materials:** "Random Number Doubles" sheets, random number generator CD/spinner/10-sided die

*The random number CD by Kim Sutton is a valuable tool for motivating students with drill and practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information.*

The random number CD can be used with any drill command using the four operations. When a number is called out, the student would perform that drill command (*double the number heard and add 2*) and record the sum. This would continue down the columns.

# Random Number ~ Doubles +2

1. double  +2 = \_\_\_\_\_

2. double  +2 = \_\_\_\_\_

3. double  +2 = \_\_\_\_\_

4. double  +2 = \_\_\_\_\_

5. double  +2 = \_\_\_\_\_

6. double  +2 = \_\_\_\_\_

7. double  +2 = \_\_\_\_\_

8. double  +2 = \_\_\_\_\_

9. double  +2 = \_\_\_\_\_

10. double  +2 = \_\_\_\_\_

11. double  +2 = \_\_\_\_\_

12. double  +2 = \_\_\_\_\_

13. double  +2 = \_\_\_\_\_

14. double  +2 = \_\_\_\_\_

15. double  +2 = \_\_\_\_\_

16. double  +2 = \_\_\_\_\_

17. double  +2 = \_\_\_\_\_

18. double  +2 = \_\_\_\_\_

19. double  +2 = \_\_\_\_\_

20. double  +2 = \_\_\_\_\_

21. double  +2 = \_\_\_\_\_

22. double  +2 = \_\_\_\_\_

23. double  +2 = \_\_\_\_\_

24. double  +2 = \_\_\_\_\_

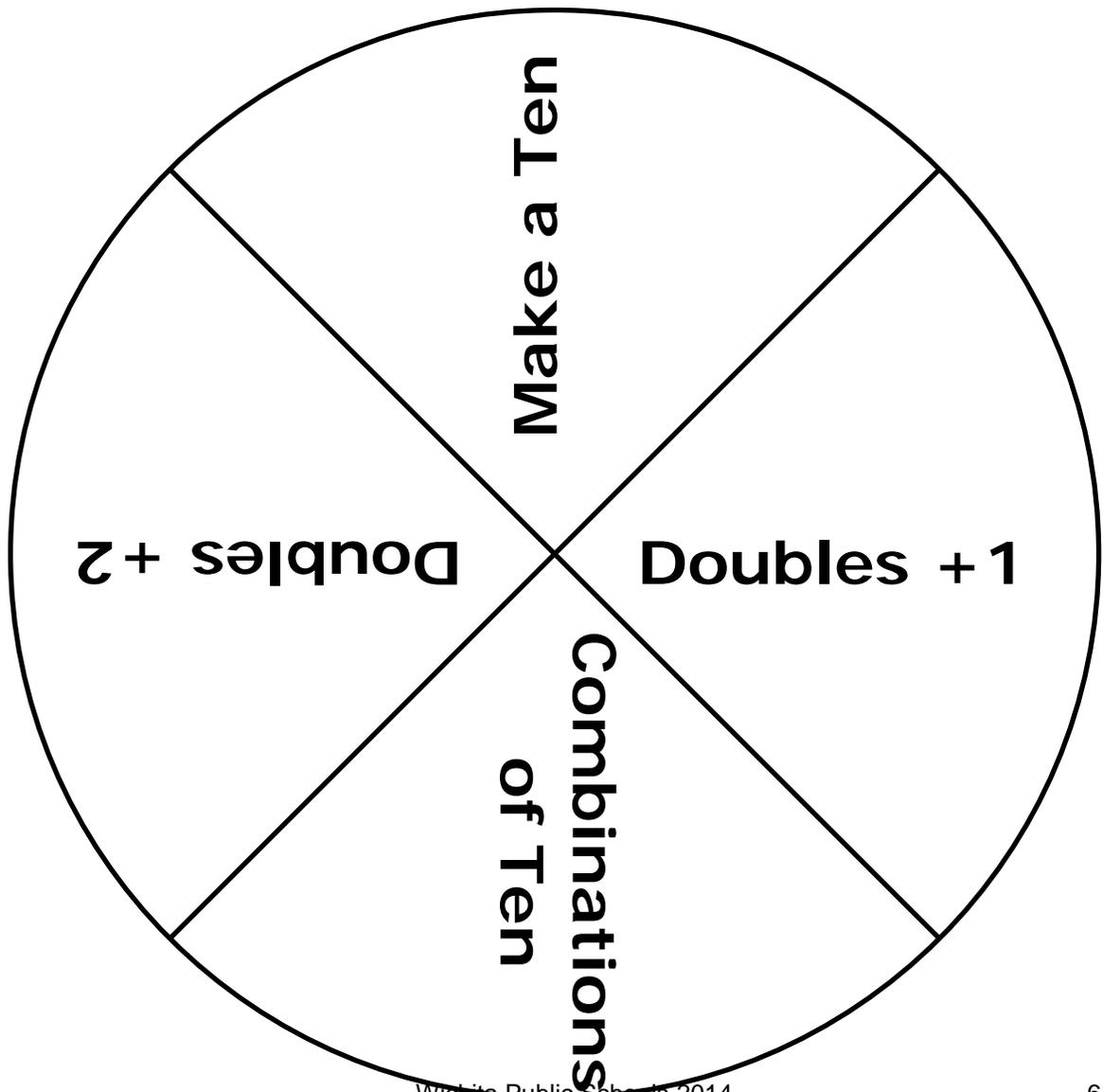
# Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "Doubles," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



# Strategy Focus Review

(Doubles +1, Combinations of Ten, Make a Ten, Doubles +2)

$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +6 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +1 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +5 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +6 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +5 \\ \hline \end{array}$

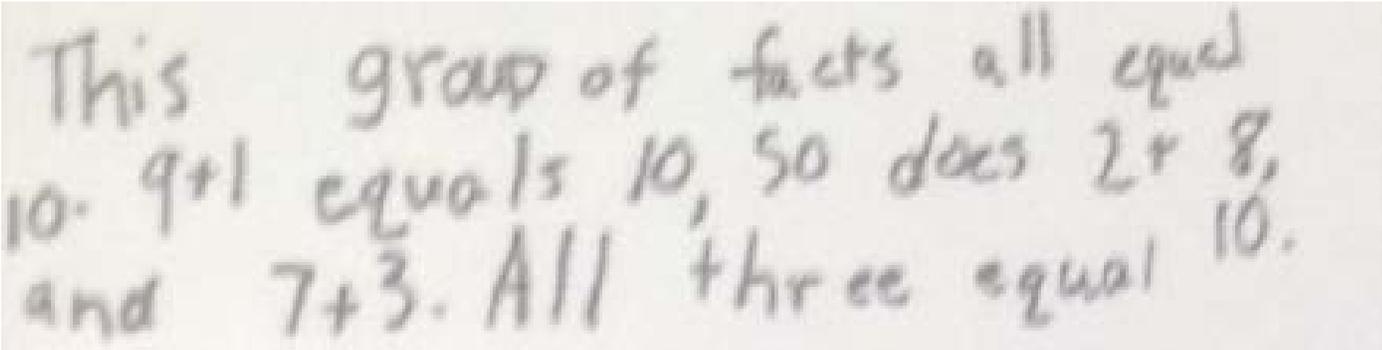
# Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 x 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts were chosen to be solved with that strategy.

Here is a student example of their reasoning – although this is with an addition strategy, you can see the thinking the student went through in using a strategy to solve the facts.



This group of facts all equal 10.  $9+1$  equals 10, so does  $2+8$ , and  $7+3$ . All three equal 10.

## Strategy Sort Mat

<b>Doubles +1</b>	<b>Combinations of Ten</b>
<b>Make a Ten</b>	<b>Doubles +2</b>
<p>Choose one strategy and explain why those facts are solved with the chosen strategy:</p>	

## **+9 Strategy (add ten take one away)**

Students will be working on the facts that have 9 as an addend.

### **Materials Needed:**

- Hundred chart for each student and teacher
- Transparent counter for each student and teacher
- Deck of cards, 10-sided die, or 1-9 spinner for the teacher

*This strategy can only be used once students are comfortable with adding 10 and then counting back 1.*

### **Teaching Fact Activity:**

#### **Materials:**

We are going to use problems to help us understand adding **nine**. Let's talk about the following problems together. (These problems can be written on the whiteboard or on chart paper.)

The principal had 7 desks in the hallway. The teacher put 9 more in the hallway. How many desks are sitting in the hallway?

» *Have students discuss the situation with their peers. Have them illustrate the problem. It may be helpful for some students to act it out.*

The baker had 9 cookies in the tray. His helper put 8 more on the tray. How many cookies does he have now?

» *Repeat the discussion.*

If needed, continue with more situations that you create and have discussions.

#### **Strategy:**

#### **Add 10 and take away 1 (+9)**

Give each student a hundred chart and a transparent counter. You have the same supplies plus a 10-sided die (or a deck of cards or a 1-9 spinner). Place your supplies under the document camera.

Explain that they will be exploring a new strategy using the hundred chart.

Tell the students to place the hundred chart in front of them and put the transparent counter at the top of the hundred chart (telling them to put the counter on the **word** Hundred will make sure they get it off the numbers in the chart).

Now, put your counter on the number 8 in your chart.

If I add 10 where does my counter land. How did you get your counter there?  
(*Students should remember the adding 10 strategy.*)

Take one away from 18 and where does your counter land?

How many spaces are between 8 and 17? (A few *students may need to count the squares but all should determine there is a difference of 9 between the two numbers.*)

Let's try another number. Start at the 6 and add 10 and take away 1. How many spaces are between 6 and 15? (If the students need more examples, explore more situations.)

If the students haven't already told you, make the observation that each time there were 9 spaces between the numbers. So we can move down on row (+10) and then back up one (-1) to add 9 or we can jump up nine spaces on our hundred chart (+1+1+1+1+1+1+1+1+1). Ask students to talk to their partner and decide which way is more efficient – counting up nine or adding 10 and taking away 1.

Have a whole class discussion about the effectiveness and efficiency of adding 10 and taking away 1. Have them practice with their counters on the hundred chart with other numbers and this time write the equations up on the board.

You will also want to show the students how this looks on the number line.

## Practice Fact Activity

### Hundred Chart Pattern

**Materials needed:** *each student will need a hundred chart, transparent counter, and a recording sheet (**Add 10 and Take Away 1**); each pair of students will a 10-sided die*

Students will use the hundred chart and the transparent counter to show the movement of adding 10 and taking away 1 when using the +9 strategy.

1. One student will roll the 10-sided die for the addend that is missing in the problem on the recording sheet. (Example: student rolls a 7 so a 7 is written in the first box.)
2. This student will then add 9 to what they rolled but they will use the add 10 and take away 1 strategy by showing the movement of the counter on the hundred chart.
3. The same student will write the sum in the correct blank for the solution.
4. The die passes to their partner and play continues until the recording sheet is completed.

## Practice Fact Activity

### **Adding Nine the on Number Line**

**Materials needed:** *each student needs a recording sheet (**Adding Nine on the Number Line**), a deck of cards or 10-sided die, and pencil (the recording page can also be laminated and used with a dry-erase marker)*

1. Draw a card or roll the die to create the missing addend in the problem below the number line. Write that number in the equation and circle it on the number line.
2. Now add nine by jumping up 10 and then backing down 1. Show this on the number line.
3. Write the sum in the equation.
4. Have a partner check your work to make sure your answers are correct and your strategy is drawn correctly on the number line.

## Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

## Add 10 and Take Away 1 (+9 strategy)

Roll a 10-sided die to fill in the missing addend (*dotted line box*). Add 9 using your hundred chart and transparent counter to show the strategy. Write the sum in the correct box (*solid line box*).

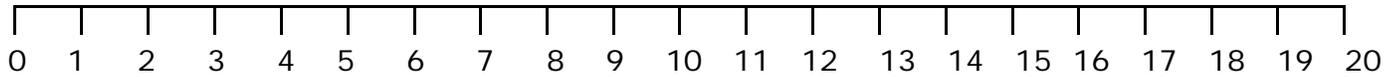
<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	+	9	=			
9	+	<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	=			
	+	9	=			
	=	9	+			
	=	<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	+	9		
	+	9	=			
	=	9	+			
	+	9	=			
9	+	<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	=			

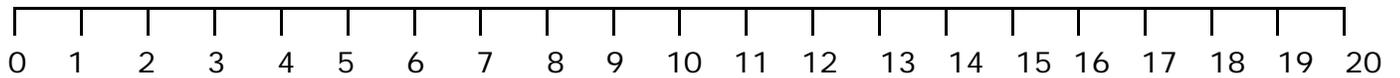
	=		+		=		9
9	+		=		=		
	=	9	+		=		
	+	9	=		=		
	=	<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	+	9	=		
	+	9	=		=		
	=	<div style="border: 1px dashed black; width: 40px; height: 40px; margin: 0 auto;"></div>	+	9	=		
9	+		=		=		
	=	9	+		=		

## Adding Nine on the Number Line

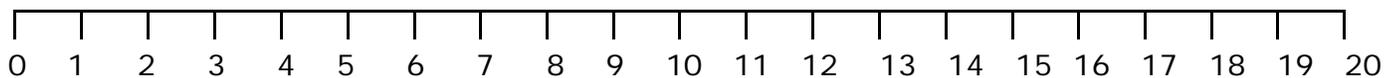
Draw a card or roll a die to create the missing addend. Write that number as the missing addend. Show how adding 9 can be done by jumping up 10 and then backing down 1 on the number line. Write the sum in the equation. Have a partner check your number line work and your equation.



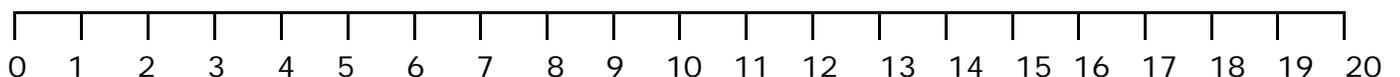
$$\square + 9 = \square$$



$$\square = 9 + \square$$



$$9 + \square = \square$$



$$\square + 9 = \square$$

## **+ 4 Strategy (+2 and +2)**

Students will be working on the facts that have 4 as an addend.

### **Materials Needed:**

- Number Line for each student and for the teacher
- Deck of cards, 10-sided die, or 1-9 spinner for the teacher

*This strategy can only be used once students are comfortable with adding 2 fluently.*

### **Teaching Fact Activity:**

#### **Materials:**

We are going to use problems to help us understand adding **four**. Let's talk about the following problems together. (These problems can be written on the whiteboard or on chart paper.)

My sister has 7 books sitting on her shelf. My aunt gave her 4 more for her birthday. How many books does my sister have?

» *Have students discuss the situation with their peers. Have them illustrate the problem. It may be helpful for some students to act it out.*

It took 4 hours to build my school project. It took my best friend 8 hours to build her project. How many hours did it take us both to finish our projects?

» *Repeat the discussion.*

If needed, continue with more situations that you create and have discussions.

#### **Strategy:**

#### **Add 2 and Add 2 (+4)**

Give each student a number line (either a series of number lines or one that can be used with dry-erase markers for repeated use) to show how they will add 2 and then add 2.

Explain how we have already learned the "counting on by 2" strategy. We will use this in helping us add 4 quickly. (If needed, you may need to review a couple of +2 problems.)

Under the document camera place a number line (again either numerous number lines or one that can be wiped off) and start with the number 7. Ask the students, what is 2 more than 7? Then what is 2 more after that? Show this on the number line.

Continue with different numbers at the starting position and have the students work them. When they become familiar with the strategy then have them work on the number lines without you showing them on the document camera. Have discussions about why this strategy works for adding 4.

You will eventually be writing the equations on the board to have students solve. Remember: you must have the equal sign in different positions as well as the 4 when doing your sample problems.

### Practice Fact Activity

#### **Adding 4 the on Number Line**

**Materials needed:** *each student needs a recording sheet (**Adding 4 on the Number Line**), a deck of cards or 10-sided die, and pencil (the recording page can also be laminated and used with a dry-erase marker)*

1. Draw a card or roll the die to create the missing addend in the problem below the number line. Write that number in the equation and circle it on the number line.
2. Now add four by jumping up 2 and jumping up 2 again. Show this on the number line.
3. Write the sum in the equation.
4. Have a partner check your work to make sure your answers are correct and your strategy is drawn correctly on the number line.

### Practice Fact Activity

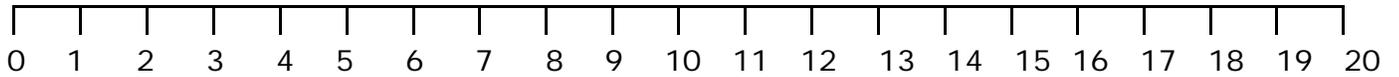
#### **Hop and Hop**

**Materials needed:** *each student will need a recording sheet (**Hop and Hop**) – this page can be laminated for reuse; each pair of students will need a 10-sided die or a deck of cards*

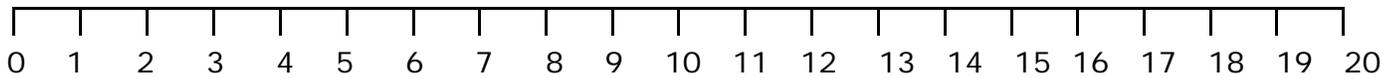
1. One student will roll the 10-sided die or draw a card from the deck to get the first number on the recording sheet. (Example: student rolls a 7 so a 7 is written in the first box.)
2. This student will then add 4 to the number but they will show the +2 and +2 strategy by showing the numbers in the circles that are used to get to the final sum.
3. The same student will write the sum in the correct blank for the solution.
4. The die or deck of card passes to their partner and play continues until the recording sheet is completed.

## Adding 4 on the Number Line

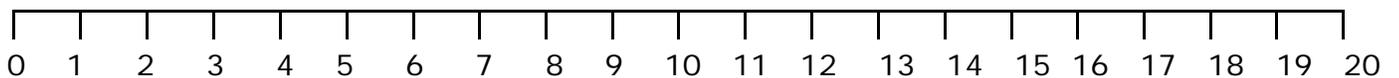
Draw a card or roll a die to create the missing addend. Write that number as the missing addend. Show how adding 4 can be done by jumping up 2 and then 2 again on the number line. Write the sum in the equation. Have a partner check your number line work and your equation.



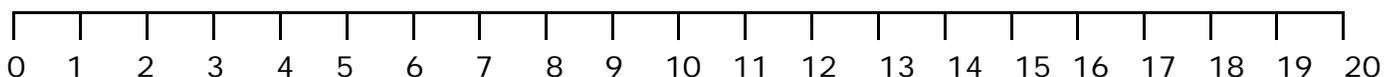
$$\square + 4 = \square$$



$$\square = 4 + \square$$



$$4 + \square = \square$$



$$\square + 4 = \square$$

## Hop and Hop (+4 strategy)

Roll a 10-sided die to fill in the missing addend (*dotted line box*). Add 4 by using the +2 and +2 strategy. Show the numbers that are used to get to the final sum by writing them in the circles. Write the final sum in the correct box (*solid line box*).

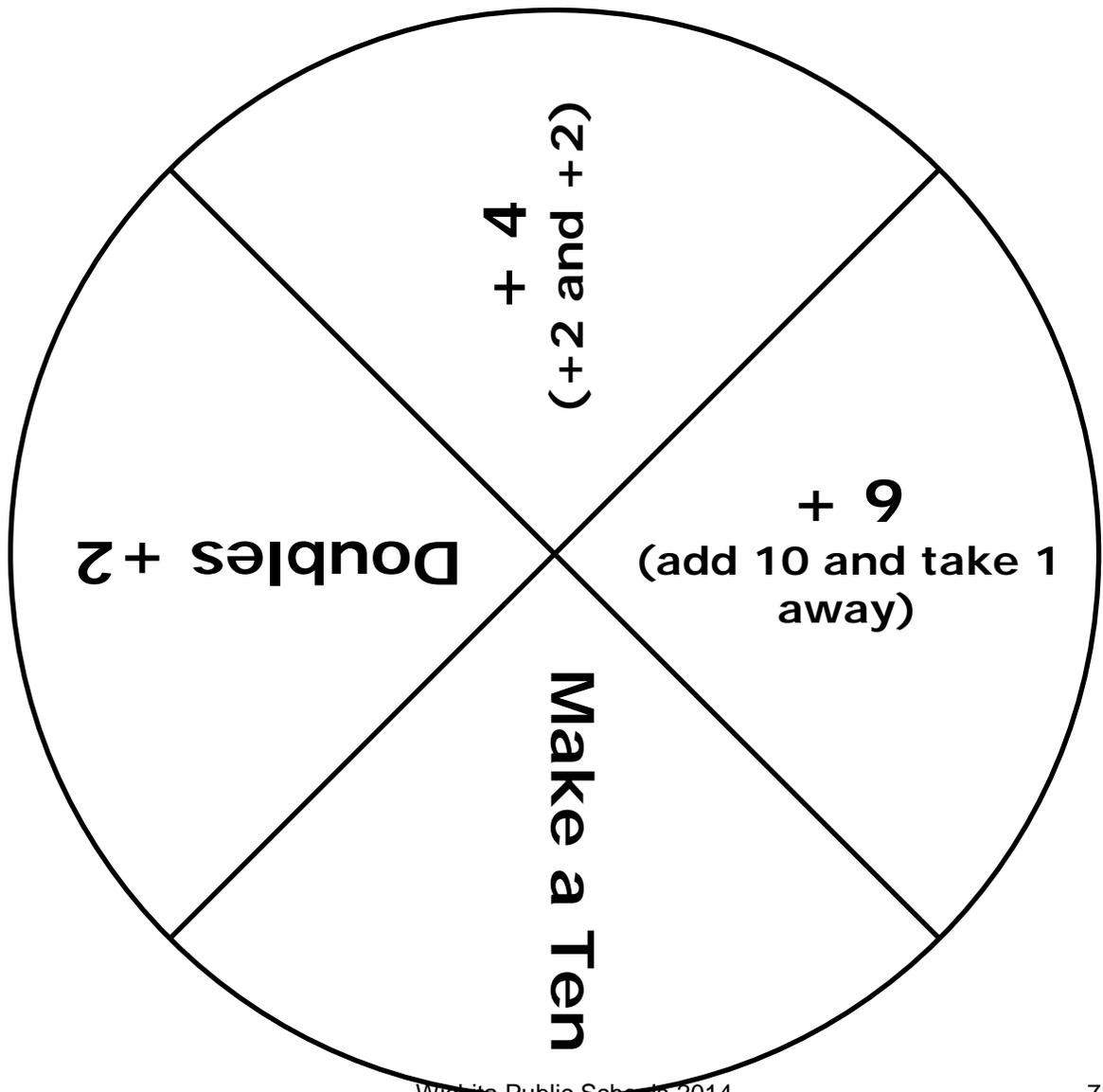
# Strategy Focus Review

Students need to practice the strategies that have been taught. They need to start reasoning out which facts are best solved with particular strategies. This should be intentionally practiced and discussed.

Use the spinner below with the facts on the next page. Give each student a copy of the facts page. Place the spinner under the document camera and spin it. The spinner will indicate which strategy the students will focus on for their facts page. If the spinner lands on "+9," then the students will identify the facts on their page that can easily be solved using that strategy and circle them.

After a short amount of time, place a copy of the facts page under the document camera and have the students tell you which facts were circled. Have students explain why that fact can be solved with that strategy for the first 3 or 4 given to you.

When facts have been circled and verified that they can be solved using that particular strategy, have the students answer the problems that are circled using the indicated strategy. Have students pair up when finished to check their answers to see if they agree.



# Strategy Focus Review

(Make a Ten, Doubles +2, + 9, + 4)

$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +4 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +5 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +9 \\ \hline \end{array}$
$\begin{array}{r} 7 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +6 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$

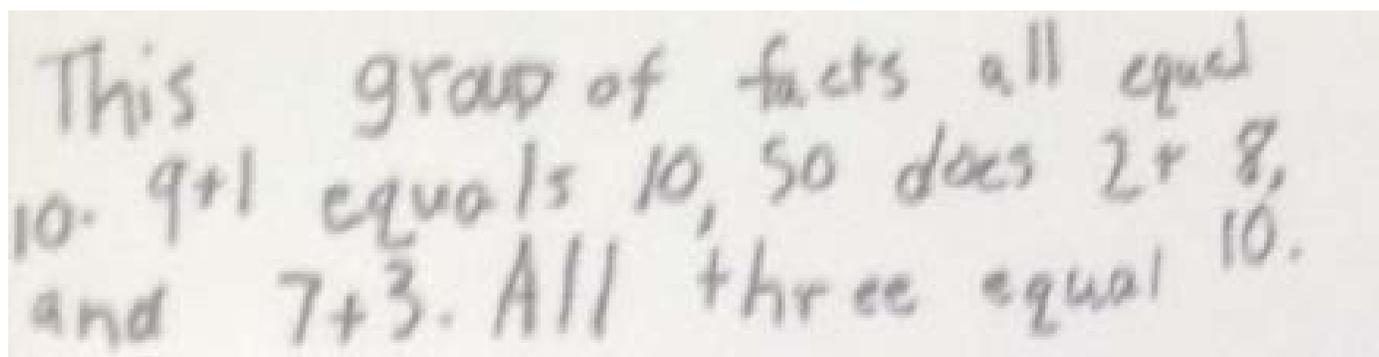
# Strategy Focus Review

Students need to be able to discuss and write about their reasoning with strategies. This activity continues the focus on strategies but engages the students cognitively by writing about their reasoning for a particular strategy for a specific fact or set of facts.

Use the previous page to create a set of cards for each student or pair of students. These cards will be sorted on the strategy mat on the following page. After the cards have been sorted, the students will check their thinking with a partner and then discuss any discrepancies between them. The students need to explain to their partner why they choose that particular strategy that may be different from their partner. This doesn't mean their strategy use was incorrect (because one fact could be solved by more than one strategy) but listening to another's reasoning could let them see another way to attack problems.

Students may switch facts around while they are talking with their partners or they may decide to leave them. The facts are finally written, taped or pasted onto the chart (if students are taping or pasting you will want to enlarge the chart onto 11 x 17 paper to allow for the cards and the writing at the bottom) and then one strategy is chosen to explain why those facts were chosen to be solved with that strategy.

Here is a student example of their reasoning – although this is with an addition strategy, you can see the thinking the student went through in using a strategy to solve the facts.



This group of facts all equal 10.  $9+1$  equals 10, so does  $2+8$ , and  $7+3$ . All three equal 10.

## Strategy Sort Mat

Make a Ten	Doubles +2
<b>+9</b> (add 10 and take 1 away)	<b>+4</b> (add 2 and add 2)
Choose one strategy and explain why those facts are solved with the chosen strategy:	

# Subtraction Readiness Assessment

## Directions and Analysis

Teaching Student-Centered Mathematics (First Edition) vol. 1 p. 107-108

Before working on mastery of subtraction facts, it is a good idea to check on students' mastery of addition facts and see if they are beginning to make connections between addition and subtraction.

### Directions for the Assessment

1. Have students number their paper 1-20.
2. Explain that you are simply trying to find out what they know so that you can help them. This is not a test or quiz. They should do their best to write what they think the answers are without counting.
3. Show each fact on the document camera one at a time for about 3 seconds each.
4. Students will write their answers as they go. They may not go back to facts they missed. *(Keep the pace quick so that students will not have a chance to count on their fingers.)*
5. Turn in papers.

### Analysis of the Assessment

If students know the addition facts but not the subtraction facts, more effort should be placed on developing the addition–subtraction connection. Use missing-part activities and join-type story problems in which the change amount is unknown. Discuss how both an addition sentence and a subtraction sentence can be written for these situations. Be very overt in your own modeling of think-addition. For  $9 - 4$  you might say, "I know that 4 and 5 make 9, so 9 minus 4 is 5." Draw a part-part-whole picture to go with this explanation.

This simple diagnostic exercise can be repeated at various times with other pairs of addition and subtraction facts to see how well students are connecting addition and subtraction. This same approach can be used in a diagnostic interview format. (See Teaching Student-Centered Mathematics [First Edition] vol. 1 p. 108)

$3 + 7 =$

$4 + 3 =$

$9 - 4 =$

$5 + 0 =$

$12 - 5 =$

$5 + 9 =$

$10 - 3 =$

$8 - 4 =$

$2 + 9 =$

$4 + 5 =$

$14 - 9 =$

$5 + 1 =$

$9 - 2 =$

$5 - 5 =$

$6 - 1 =$

$7 + 5 =$

$7 - 4 =$

$11 - 2 =$

$7 + 2 =$

$4 + 4 =$

## Subtraction Strategy

# Think Addition

### Materials Needed:

- Word problems (under document camera or on chart paper)
- subtraction/addition fact cards (each)

*Subtraction facts prove to be more difficult than addition. This is especially true when children have been taught subtraction through a “count-count-count” approach. When subtraction is modeled in a way that students are encouraged to think, “What goes with this part to make the total?” the child will use known addition facts to produce the unknown quantity or part. If this important relationship between parts and wholes—between addition and subtraction—can be made, subtraction facts will be much easier. Word problems that promote think-addition are those that sound like addition but have a missing addend: Add to, start unknown; Add to, change unknown; and Put together/Take apart, addend unknown. (VDW 106-107)*

### Teaching Fact Activity:

**Materials:** paper & pencil or whiteboard, document camera

We are going to use problems to help us understand the “Think-Addition” strategy. Let’s talk about the following problems together.

Sid has some gum. He gave 4 pieces to his friend. He has 6 pieces left. How many pieces of gum did he start with? *(Taken from, start unknown)*

» *Have students discuss with a partner and with the entire group the thinking that was used to solve the problem. Eventually you can have students illustrate the problem or show it on a number line. It may also be helpful for some students to act it out.*

Sara has 8 pencils. Mike has 4. How many more does Sara have than Mike? *(Compare, difference unknown)*

Brian has 13 cubes. He has 7 more than Lindsay. How many does Lindsay have? *(Compare, smaller unknown)*

Paul’s grandmother have him 16 cookies to share. He shared some of the cookies. He has 8 cookies left in the box? How many cookies did he share? *(Take apart, addend unknown)*

Cindy has 8 carrot sticks. She gave 3 carrot sticks to her friend. How many does she have left? *(Taken from, result unknown)*

Tina has 13 stuffed animals. 7 of them are new. How many has she had for a while? (*Put together/take apart, addend unknown*)

Jill had 9 pennies. She gave some to Rick. She has 6 left. How many did she give to Rick? (*Taken from, change unknown*)

## Practice Fact Activity:

**Find a Plus Fact to Help** (Teaching Student-Centered Mathematics [First Edition] vol. 1, Activity 4.16)

**Materials:** subtraction/addition fact cards (each)

### Directions:

1. Select a group of subtraction facts that you wish to practice (a sample set is available for use).
2. If creating your own set, each subtraction should have a corresponding addition fact.
3. Have students cut their cards apart.
4. Write one of the subtraction facts on the board.
5. Rather than call out answers, students find the addition fact that helps with the subtraction fact.
6. On your signal, each student holds up the appropriate fact. For examples, for  $12 - 4$  or  $12 - 8$ , the students would select  $4 + 8$ .

\*The same activity can be made into a matching card game

$4 + 8$

$12 - 8$

$5 + 4$

$9 - 5$

$9 + 7$

$16 - 9$

$2 + 6$

$8 - 6$

$7 + 7$

$14 - 7$

$6 + 7$

$13 - 7$

$8 + 7$

$15 - 8$

$5 + 6$

$11 - 6$

$9 + 9$

$18 - 9$

$6 + 4$

$10 - 6$

## *Template*


## Subtraction Strategy

# Addition/Subtraction Related Equations

**Materials Needed:**

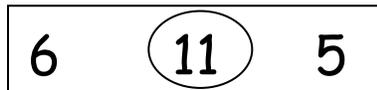
- Addition Fact Families (by addition strategy)
- Addition/Subtraction triangle flash cards

### Teaching Fact Activity:

**Missing Number Cards** (Teaching Student-Centered Mathematics [First Edition] vol. 1, Activity 4.14)

**Materials:** missing number cards

Show children, without explanation, related equations with the sum circled. See the example below:



Ask: Why do you think the numbers go together? Why is one number circled?

Record the number sentences that represent the relationship of the numbers within the Venn diagram.

Example:  $6 + 5 = 11$   
 $5 + 6 = 11$   
 $11 = 5 + 6$   
 $11 = 6 + 5$

$11 - 6 = 5$   
 $11 - 5 = 6$   
 $5 = 11 - 6$   
 $6 = 11 - 5$

When this related equations idea is fairly well understood, show some related equations with the one number replaced by a question mark. See example below:



Ask: What number is missing? How do you know?

Once students understand the activity, student will need to write an addition fact and a subtraction fact to go with each missing-number card. You may wish to have students work in small groups.

## Practice Fact Activity:

**Missing-Number Worksheets** ([Teaching Student-Centered Mathematics](#) vol.1, Act. 4.15)

**Materials:** missing-number worksheet(s), answer key

### Directions:

1. Have students fill in the missing numbers on the worksheet. The facts have been listed according to their fact strategy. Worksheets should be cut into strips in order for students to focus on specific fact strategies.
2. You may wish to assign this only one strategy at a time for review of strategies that students are comfortable with, or differentiate according to the strategies your students need additional practice with.
3. Because students do not always connect this knowledge of the missing part to addition or subtract, it is important that students write an addition and subtraction problem.
4. Answer keys are also included. These could be a nice set of miniature flash cards students could use to practice at home. 😊

## Practice Fact Activity:

**Triangle Flash Cards**

**Materials:** flash cards (each)

### Directions:

1. Triangle flash cards can be used individually, as well as in small group settings.
2. To practice addition facts, use a finger to cover the sum under the underlined number (top of the triangle).
3. To practice subtraction facts, use a finger to cover one of the addends.
4. It is best to focus on a smaller set of facts at a time. This can be done by a specific strategy, or by a chosen set of problems that are considered "hard" for the student.

## Missing Number Cards

4 ○ 5

8 9 ○

7 ○ 14

8 ○ 3

6 ○ 8

11 ○ 4

○ 3 6

○ 10 4

---

---

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## Missing Number Cards

4 ○ 5

8 9 ○

7 ○ 14

8 ○ 3

6 ○ 8

11 ○ 4

○ 3 6

○ 10 4

---

---

# Missing Number Worksheets

Counting On

$4 + \quad = 3$

$9 + 1 + \quad = \quad$

$\quad + 5 + 2 = \quad$

$\quad + 6 + 5 = \quad$

$5 + 2 + \quad = \quad$

$2 + 6 + \quad = \quad$

$6 + \quad + 1 = \quad$

$1 + \quad + 9 = \quad$

$\quad + 10 + 2 = \quad$

$7 + \quad + 1 = \quad$

$9 + 2 + \quad = \quad$

$1 + 5 + \quad = \quad$

$7 + \quad + 2 = \quad$

Zero

$9 + 0 + \quad = \quad$

$0 + \quad + 0 = \quad$

$\quad + 4 + 4 = \quad$

$7 + \quad + 0 = \quad$

$2 + 2 + \quad = \quad$

$8 + \quad + 8 = \quad$

$\quad + 0 + 3 = \quad$

$\quad + 0 + 7 = \quad$

$\quad + 0 + 5 = \quad$

$1 + 1 + \quad = \quad$

$9 + 9 + \quad = \quad$

$\quad + 0 + 6 = \quad$

$8 + \quad + 8 = \quad$

Doubles

$\quad + 18 + 9 = \quad$

$7 + \quad + 7 = \quad$

$12 + \quad + 6 = \quad$

$3 + 3 + \quad = \quad$

$10 + \quad + 5 = \quad$

$\quad + 7 + 7 = \quad$

$8 + 16 + \quad = \quad$

$8 + 4 + \quad = \quad$

$10 + 20 + \quad = \quad$

$4 + \quad + 8 = \quad$

$16 + \quad + 8 = \quad$

$\quad + 6 + 12 = \quad$

$\quad + 10 + 5 = \quad$

# Missing Number Worksheets

Doubles +1

$13 + 6 = \quad$

$4 + 5 = \quad$

$2 + \quad = 3$

$\quad + 11 = 6$

$6 + \quad = 11$

$\quad + 17 = 8$

$15 + 8 = \quad$

$4 + \quad = 7$

$\quad + 9 = 4$

$\quad + 8 = 9$

$7 + 8 = \quad$

$4 + 9 = \quad$

$7 + \quad = 3$

Combinations of Ten

$\quad + 5 = 10$

$10 + 2 = \quad$

$6 + \quad = 10$

$\quad + 1 = 9$

$7 + 10 = \quad$

$4 + 6 = \quad$

$\quad + 7 = 10$

$\quad + 1 = 9$

$10 + \quad = 8$

$\quad + 10 = 7$

$\quad + 10 = 5$

$9 + \quad = 10$

$10 + \quad = 3$

Make a Ten

$\quad + 11 = 4$

$4 + 12 = \quad$

$16 + \quad = 7$

$8 + \quad = 12$

$12 + \quad = 7$

$11 + 4 = \quad$

$\quad + 13 = 9$

$\quad + 3 = 9$

$\quad + 14 = 8$

$5 + \quad = 14$

$12 + 5 = \quad$

$7 + 9 = \quad$

$9 + 13 = \quad$

# Missing Numbers

Answer key

Counting On

4 1 3

9 1 10

3 5 2

1 6 5

5 2 7

2 6 4

6 5 1

1 8 9

8 10 2

7 6 1

9 2 11

1 5 4

7 5 2

Zero

9 0 9

0 0 0

0 4 4

7 7 0

2 2 0

8 0 8

3 0 3

7 0 7

5 0 5

1 1 0

9 9 0

6 0 6

8 0 8

Doubles

9 18 9

7 14 7

12 6 6

3 3 6

10 5 5

14 7 7

8 16 8

8 4 4

10 20 10

4 4 8

16 8 8

6 6 12

5 10 5

# Missing Numbers

Answer key

Doubles + 1

13 6 7

4 5 9

2 5 3

5 11 6

6 5 11

9 17 8

15 8 7

4 3 7

5 9 4

17 8 9

7 8 15

4 9 5

7 4 3

Combinations of Ten

5 5 10

10 2 8

6 4 10

10 1 9

7 10 3

4 6 10

3 7 10

10 1 9

10 2 8

3 10 7

5 10 5

9 1 10

10 7 3

Make a Ten

7 11 4

4 12 8

16 9 7

8 4 12

12 5 7

11 4 7

4 13 9

12 3 9

6 14 8

5 9 14

12 5 7

7 9 16

9 13 4

## Subtraction Strategy

# Build Up Through Ten

**Materials Needed:**

- Ten-frames; ten-frame flashcards; counters

*This group of facts includes all facts where the part or subtracted number (minuend) is either 8 or 9. Examples are  $14 - 9$  and  $15 - 8$ .*

## Teaching Fact Activity:

Build Up Through the Ten –Frame (Teaching Student-Centered Mathematics [edition 1] vol. 1 Activity 4.12)

1. On the board, draw a ten-frame with 9 dots (or show the 9 ten-frame flashcard under the document camera). Discuss how you can build numbers between 11 and 18, starting with 9 in the ten-frame. Stress the idea of one more to get to 10 and then the rest of the number. You may need to show with counters so they can see the full ten-frame and the rest below.
2. Repeat for a ten-frame showing 8.
3. Next, with either the 8 or 9 ten-frame in view, call out numbers from 11 to 18, and have student explain how they can figure out the difference between that number and the one on the ten-frame.
4. Later, use the same approach but show fact cards to connect this idea with the symbolic subtraction fact.

# Subtraction Strategy

## Back Down Through Ten

### Materials Needed:

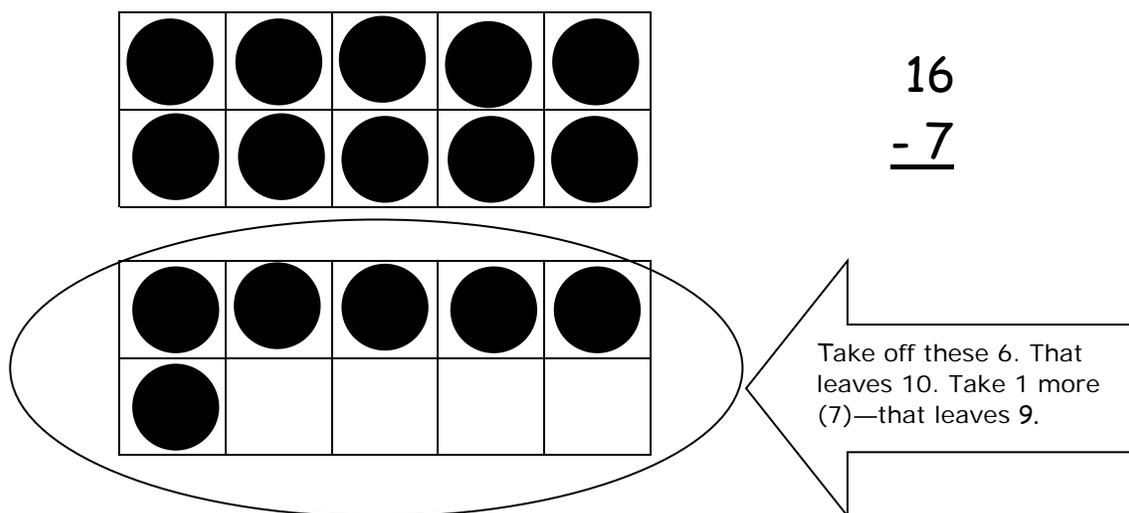
- Ten-frames

*This strategy is really a take-away and not think-addition. It is useful for facts where the ones digit of the whole (subtrahend) is close to the number being subtracted (minuend). For example, for 15-6, you start with the total of 15 and you are working backward using 10 as a “bridge” so 6 needs to be decomposed into 5 and 1. Then take 5 away to get to 10 and then one more to get the answer of 9.*

### Teaching Fact Activity:

“Back Down Through the Ten-Frame” (Teaching Student-Centered Mathematics [edition 1] vol 1. Activity 4.13)

1. Start with two ten-frames to show 16, one the complete 10 frame and the other the 6 ten frame. (see example below)



16  
- 7

Take off these 6. That leaves 10. Take 1 more (7)—that leaves 9.

2. Ask the students to tell you how the 7 can be decomposed to work with the ten frames we have in front of us. (If one student doesn't see how the 7 can be decomposed into 6 and 1 so that the 6 can be taken away first and then the one more to get the answer of 9, lead them to that thinking.)
3. For 13, as an example, discuss what is the easiest way to think about taking off 4 counters or 5 counters. Talk about the decomposition and how that works for us.
4. Repeat with other numbers between 11 and 18.
5. Have students write or say the fact as they work out the solutions.

# Practice and Review

Students now have several strategies that can help them solve addition facts. The important part now is to practice using these strategies. Be sure to give students the opportunity to communicate which strategies they find are most helpful and efficient. You may choose to repeat or continue any activities that have been previously covered that focus on a particular set of facts, or you may choose any of the following activities that will give students the opportunity to practice these strategies with numerous facts. Your decision should be based upon what you see your students need.

Additional information and suggested activities can be found in [Teaching Student-Centered Mathematics, First Edition \(Volume I\)](#) by John Van de Walle, Chapter 4, pages 94 – 111.

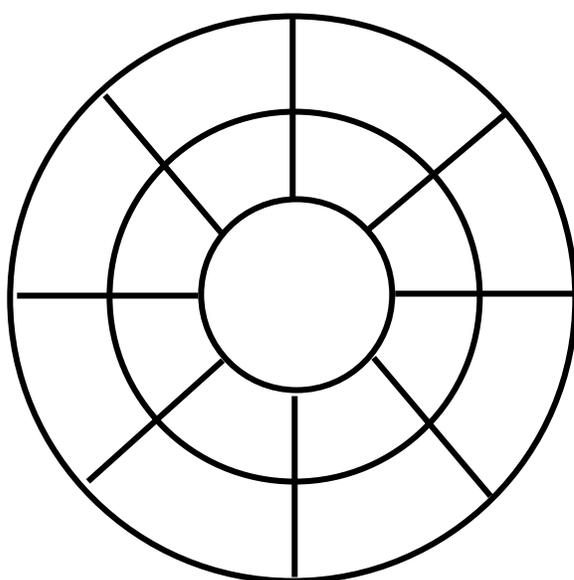
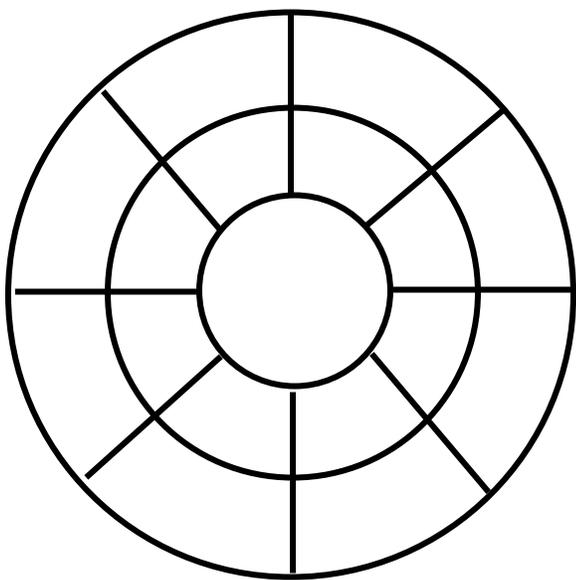
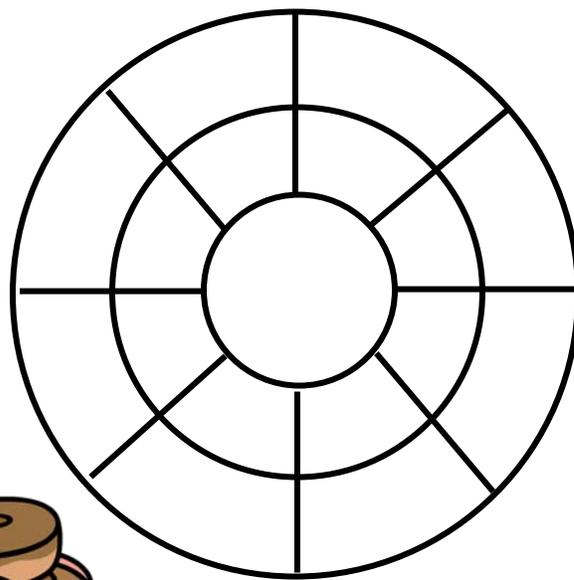
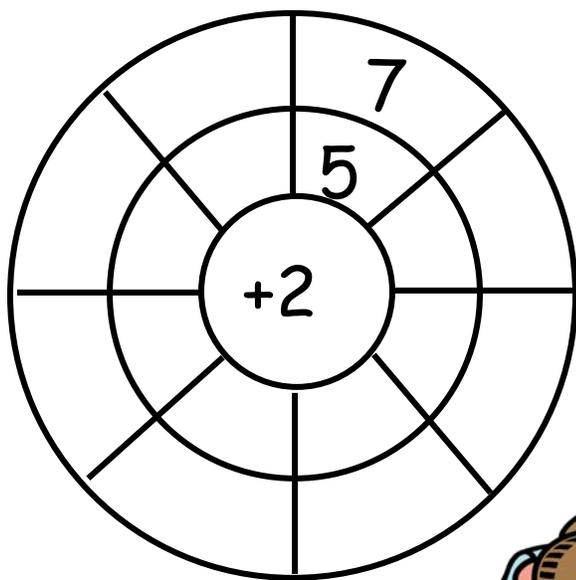




\_\_\_\_\_ Name

# Drill Doughnuts

The drill command is written in the center of the doughnut. Roll a die. Write the number in the middle section, of the doughnut. Then add that number to the drill command to get the answer.



# Random Number Generator

## **Materials:**

- Drill Command sheets,
- random number generator CD/spinner/10-sided die

## **About the Activity:**

*The random number CD is a valuable tool for motivating students with drill and practice. The random number CD is designed to generate the digits 0-9 with background music. This helps students learn to filter out unnecessary sounds and listen for important information. The first few songs are read at 8-second intervals, the last few are read at 5-second intervals.*

The random number CD can be used for early learners to practice recording the numbers that are heard. This is an excellent assessment tool for auditory number recognition.

The random number CD can be used with any drill command using the four operations. For instance, if the drill command were +6, then students would be working on this set of basic addition facts:

$0+6=6$	$5+6=11$
$1+6=7$	$6+6=12$
$2+6=8$	$7+6=13$
$3+6=9$	$8+6=14$
$4+6=10$	$9+6=15$

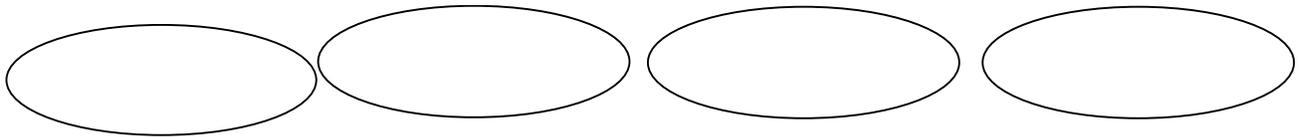
The random number CD can be used with a Drill Command practice sheet and an identified drill command to assess the students' competency with any set of facts. The student would record the drill command at the top square of the grid paper. When a number is called out, the student would perform that drill command and record the sum or product. This would continue down the columns. Mixed drill commands could also be given for each column.

*Adapted from K. Sutton materials*

Name \_\_\_\_\_

Date \_\_\_\_\_

# Drill Command Practice Sheet



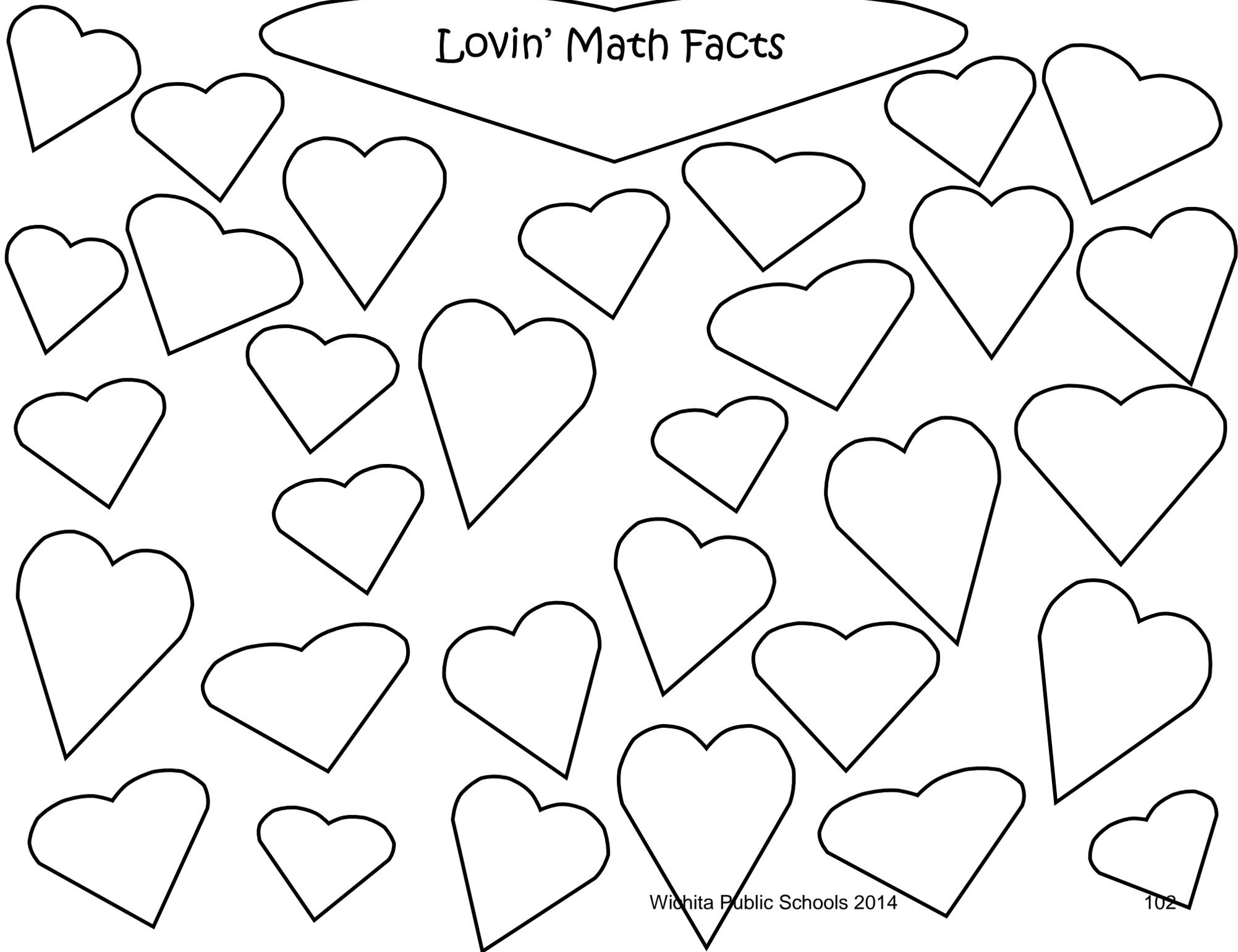
Drill Command

Drill Command

Drill Command

Drill Command

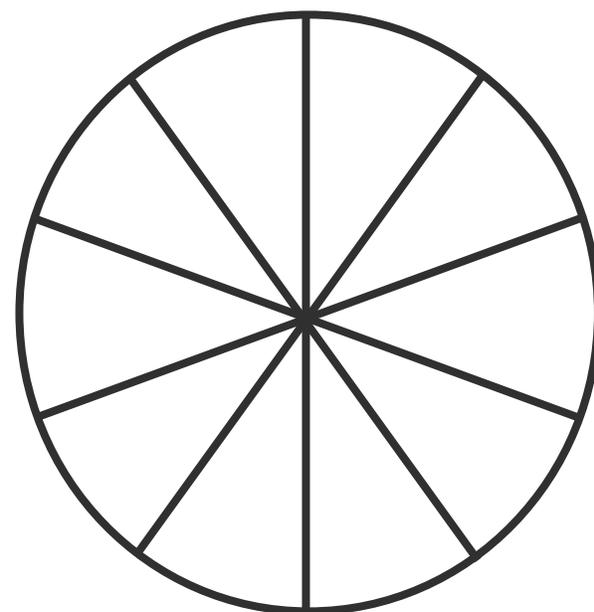
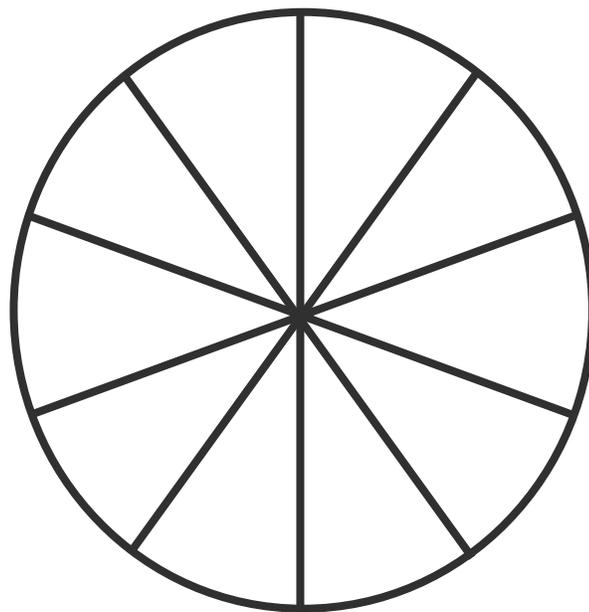
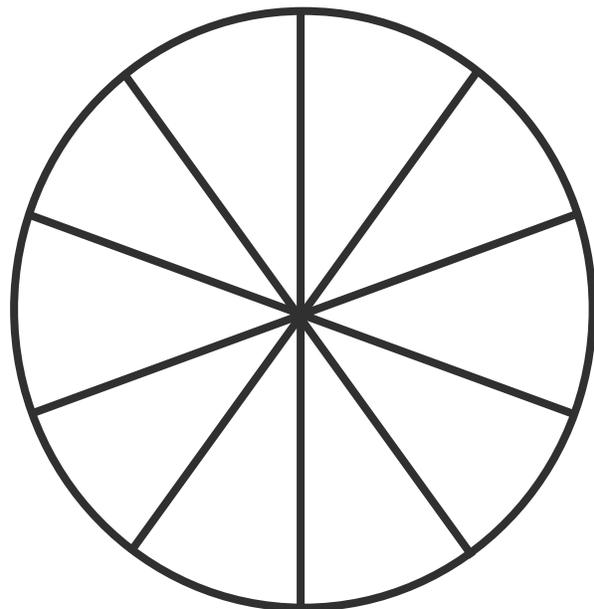
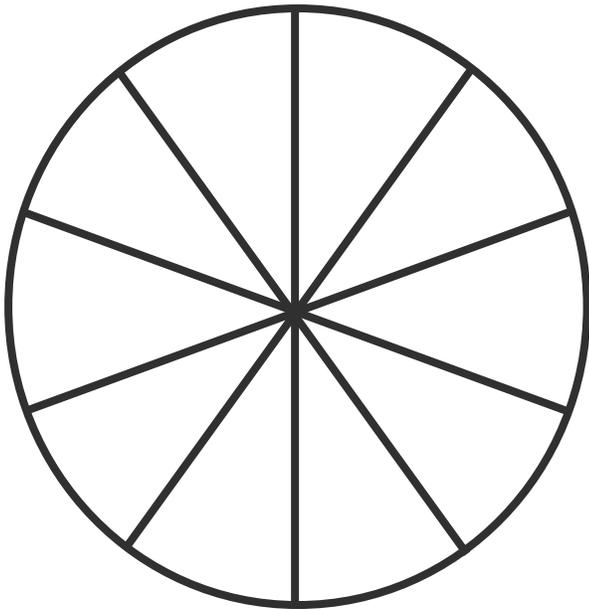
# Lovin' Math Facts



# Spinners

## Random Number Generator

Put numbers in the spaces of the spinner. Place a paper clip over the center of the spinner and the point of a pencil at one end of the paper clip and at the center point of the spinner. Flick the paper clip to make it spin.



# Number Strips

## Random Number Generator

Each student would have their own number strip to practice a specific strategy. The student will lay the number strip beside their paper and write the answer for the drill command beside it. Making larger versions for the chalkboard are also very motivational for students to practice their facts.

9
4
3
2
8
5
1
7
6
8

5
7
3
6
4
1
9
8
2
5

3
6
7
4
9
8
5
1
2
8

7
6
4
8
2
3
5
1
9
4

1
6
8
7
6
2
3
4
8
9

# Knock Your Block Off

## Addition

### Materials:

- Game boards (black lines onto Bristol paper and laminate)
- Two 10-sided die
- Connecting cubes to use for game pieces (or coins)

### Directions:

1. Two to three players may play on the same gameboard.
2. Give player one a set of 10 cubes in one color. Player two receives a set of 10 cubes but in a different color. If there is a third player, they get a set of cubes in a different color from the other two players.
3. The first player rolls the two number cubes and adds them together.
4. If they answer the problem correctly, the player places one of his colored cubes onto that answer on the gameboard. (E.g.:  $3 + 4 = 7$ , so a colored cube is placed on the 7 on the board). If they answer the problem incorrectly, the player doesn't place a cube onto the board.
5. Play goes to the next player and continues the same way.
6. When a player rolls a problem whose answer is covered by another player's cube, the player that rolled can "Knock Their Block Off" by removing their opponent's cube and placing their own in its place.
7. The first player to get 5 of their colored cubes in a horizontal, vertical, or diagonal row – WINS.

# Knock Your Block Off

<b>14</b>	<b>6</b>	<b>18</b>	<b>16</b>	<b>9</b>
<b>13</b>	<b>11</b>	<b>17</b>	<b>10</b>	<b>12</b>
<b>15</b>	<b>8</b>	<b>13</b>	<b>14</b>	<b>3</b>
<b>16</b>	<b>17</b>	<b>14</b>	<b>18</b>	<b>16</b>
<b>12</b>	<b>7</b>	<b>1</b>	<b>15</b>	<b>14</b>

# Knock Your Block Off

<b>18</b>	<b>11</b>	<b>9</b>	<b>12</b>	<b>10</b>
<b>16</b>	<b>15</b>	<b>14</b>	<b>8</b>	<b>17</b>
<b>12</b>	<b>13</b>	<b>4</b>	<b>0</b>	<b>16</b>
<b>9</b>	<b>17</b>	<b>13</b>	<b>10</b>	<b>15</b>
<b>8</b>	<b>12</b>	<b>16</b>	<b>9</b>	<b>1</b>

# Knock Your Block Off

<b>12</b>	<b>6</b>	<b>10</b>	<b>9</b>	<b>18</b>
<b>11</b>	<b>8</b>	<b>13</b>	<b>15</b>	<b>17</b>
<b>14</b>	<b>18</b>	<b>12</b>	<b>11</b>	<b>16</b>
<b>9</b>	<b>16</b>	<b>14</b>	<b>8</b>	<b>10</b>
<b>13</b>	<b>5</b>	<b>13</b>	<b>10</b>	<b>3</b>

# Knock Your Block Off

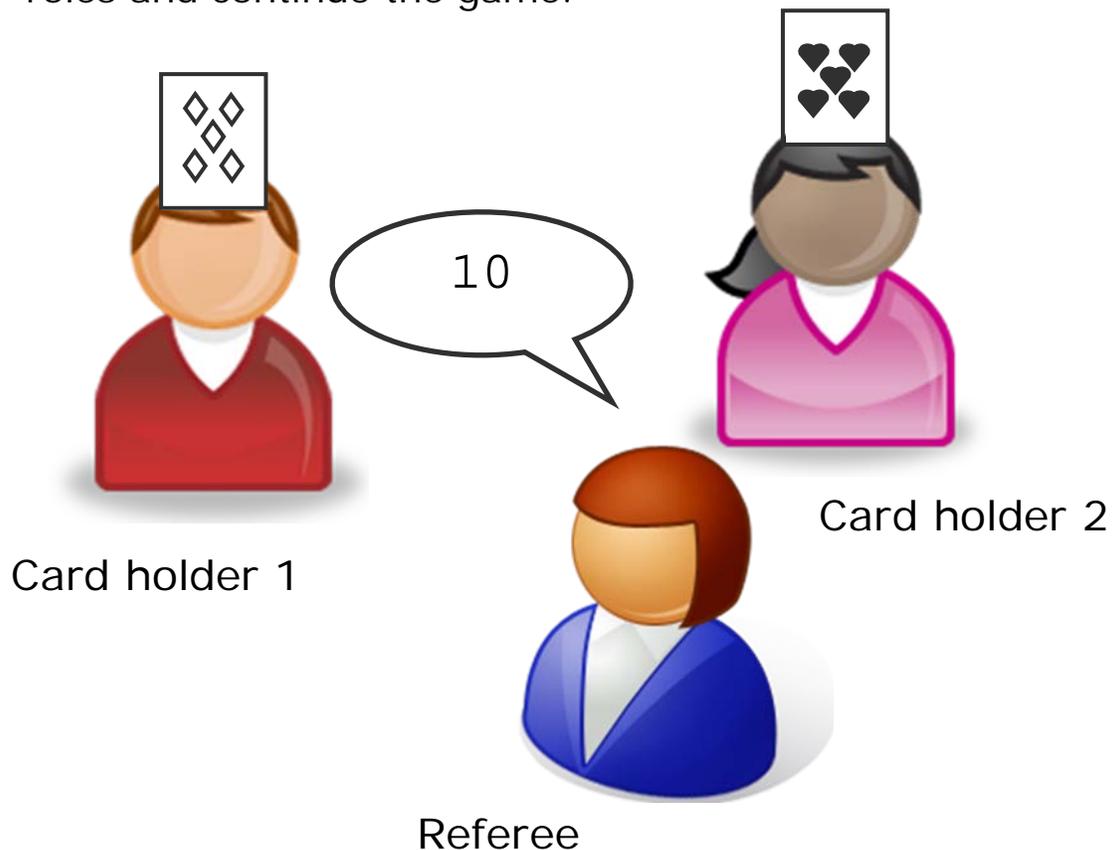
<b>4</b>	<b>9</b>	<b>10</b>	<b>12</b>	<b>17</b>
<b>12</b>	<b>8</b>	<b>11</b>	<b>13</b>	<b>15</b>
<b>16</b>	<b>14</b>	<b>18</b>	<b>5</b>	<b>2</b>
<b>13</b>	<b>15</b>	<b>9</b>	<b>10</b>	<b>8</b>
<b>1</b>	<b>7</b>	<b>13</b>	<b>18</b>	<b>16</b>

# Salute!

Materials Needed: deck of cards, or numbers written on 5 + 7 cards

## Directions:

1. This activity needs 3 students to participate. One student is the referee, and the other 2 are the card holders. These 2 people face each other.
2. Each card holder draws a card from the deck without looking at the number on it. When the referee says, 'Salute,' each card holder places the card on his or her forehead (without looking).
3. The referee tells the 2 card holders the sum, looking at the other person's card, each card holder should determine the value of their card.
4. Once both numbers are determined, students change roles and continue the game.



# Math War

Players: 2

Materials: deck of playing cards or digit cards

Directions:

1. Deal out all the cards to each player. The players place the stack of cards face down in front of them.
2. When the dealer says "Go", the players will take the first 2 cards at the top of their stack and turn it face up next to their stack of cards.
3. All the players look at their cards and whoever has the largest sum gets all of the cards that are face up.
4. If the players have cards that have the same sum, then a war takes place. The players with the matching sum take the next 2 cards and add those. Whoever has the largest sum with the 2 new cards wins those cards plus the cards from the previous hand.

Optional version: *Players will play for the smallest sum instead. Whoever has the smallest sum will win the cards.*



# Bug Ya!



## Materials:

- 2 cubes
- Container (box, can, etc.)
- Scratch paper for scoring

**Procedures:** Put a bug on one side of each cube. (You can draw a bug or use a bug sticker) Put the numbers 2 and 5 on opposite sides, put 3 and 4 on opposite sides, and put 1 opposite the bug. (Note: you will have to use larger numbers for the larger addition facts.)

**Instructions:** Each child takes turns rolling the set of cubes. The student keeps a running score of his total.  
If one bug comes up, the child subtracts the number from the total score.  
If both bugs appear, the student loses the total score.  
Use 2 cubes for each couple playing.  
First child to reach 100 is the winner!

## Example:

3	6	ADD 9	$\begin{array}{r} 9 \\ +9 \\ \hline 18 \\ -2 \\ \hline 16 \\ -16 \\ \hline 0 \end{array}$
4	5	ADD 9	
2		SUBTRACT 2	
		LOSE TOTAL SCORE	

Concepts: Reinforce addition and subtraction. (can be used for multiplication by multiplying cubes instead of adding)

Younger children can play this by keeping track of their core with quiet counters. After 3 times around, each child counts the number of counters he has. The one with the most is the winner!

# Sneaky Snake

## Materials needed:

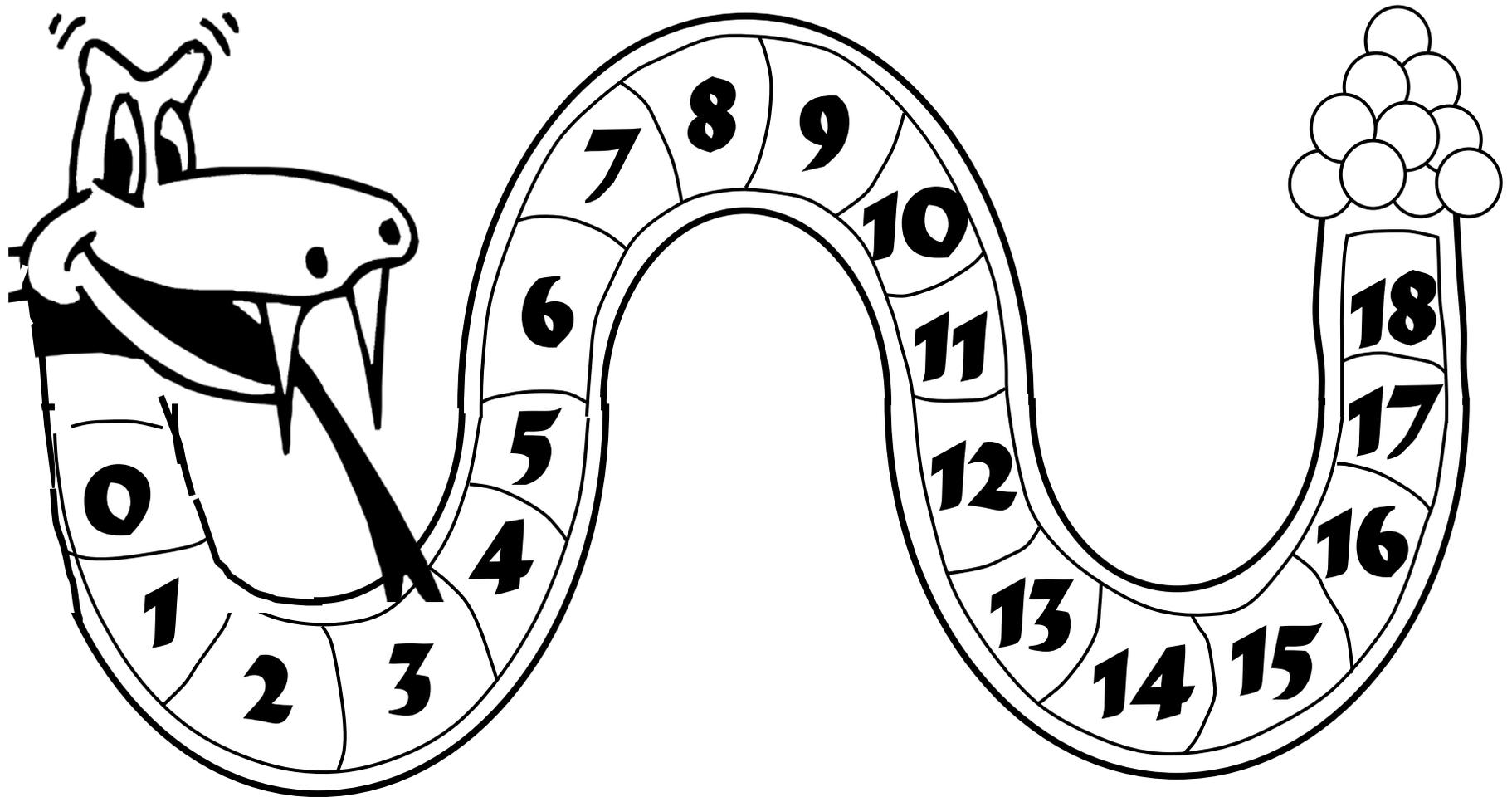
- Game board for each player (can be printed on different colors of Bristol and laminated)
- Two 10-sided dice
- Counters or cubes for each player as markers to cover up the numbers on the game board

## Directions:

1. Each player needs a game board and some counters or cubes as markers for the game board.
2. The youngest player will begin the game and play will continue clockwise from that player.
3. All players take turns by rolling the dice and covering up either the sum or the difference on their game board.
4. When a player rolls the dice and the sum and the difference has already been covered on the game board, the dice passes to the next player.
5. The first player to cover the entire board is the WINNER!

# Sneaky Snake

Each player gets a game board. Two 10-sided dice are rolled and the player either adds or subtracts the numbers. The sum or difference is covered on their game board. The player who covers their board first – WINS!



# Crack the Code

1	2	3
4	5	6
7	8	9

1.  $\square + \square =$

2.  $\square + \square =$

3.  $\square + \square =$

4.  $\square + \square =$

5.  $\square + \square =$

6.  $\square + \square =$

7.  $\square + \square =$

8.  $\square + \square =$

9.  $\square + \square =$

10.  $\square + \square =$